Why Do Borrowing Conditions for Sovereign Debt Differ?

Lender's Trust And Borrower's Cost

Yanling Guo

Goethe University Frankfurt

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Outline

Subject of this paper

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- Collateralized vs. non-collateralized debt

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- Private vs. public lenders

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- Public vs. publicly garanteed vs. private debt

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- Public vs. publicly garanteed vs. private debt
- Domestic vs. external debt?

Domestic vs. external debt

classification criterion	**Domestic Debt**	**External Debt**
citizenship of lenders	residents	foreigners
juridical governance	domestic law	foreign law
currency denomination	domestic currency	foreign currency

Domestic vs. external debt

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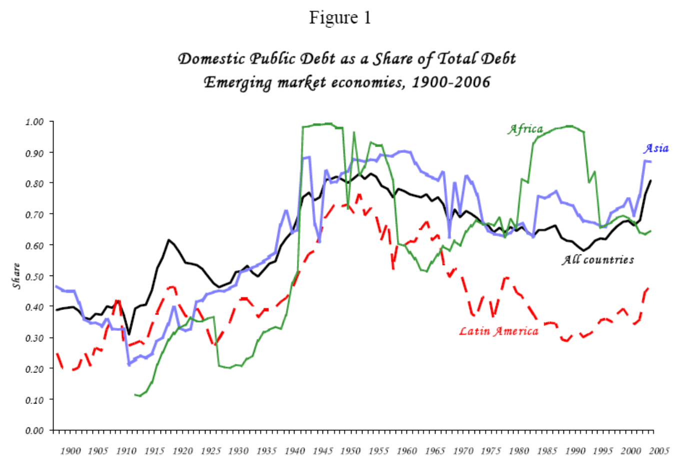
Empirical evidence

"Unambiguously, output declines in the run-up to default on domestic debt are typically significantly worse than for external debt. As ..., the average cumulative decline in output during the three-year run-up to a domestic default crisis is 8 percent. The output decline on the year of the domestic debt crisis alone is 4 percent; the comparable average decline for the external debt events is 1.2 percent."

Reinhart and Rogoff (2011)

Empirical evidence



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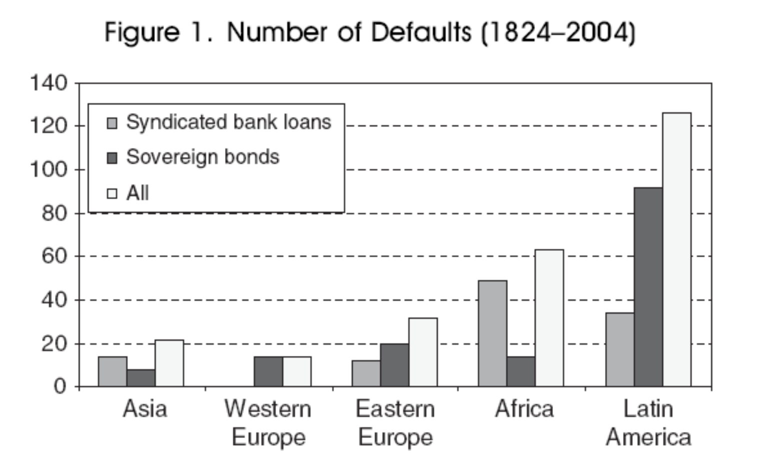

Borensztein and Panizza (2009)

Model comparison

	Eaton&Gersovitz	Calvo
Does tax cause dead-weight loss?	No	Yes
And exclusion from the financial market?	Yes	No
Default cost depends on defaulted amount?	No	Yes
Is domestic or external debt modeled?	external*	domestic*

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Models of default probability

- Eaton,Jonathan and Mark Gersovitz, "Debt with Potential Repudiation: Theoretical and Empirical Analysis", Review of Economic Studies, April 1981,48 (2), 289-309.
- Aguiar, Mark and Gita Gopinath, "Defaultable debt, interest rates and the current account", Journal of International Economics, June 2006, 69 (1), 64-83.
- Arellano, Cristina, "Default Risk and Income Fluctuations in Emerging Economies", American Economic Review, 2008, 98 (3), 690-712.

Models of default rate

- Calvo, Guillermo A., "Servicing the Public Debt: The Role of Expectations", The American Economic Review, 1988, 78(4), pp. 647-661.
- Uribe, Martin, "A fiscal theory of sovereign risk", Journal of Monetary Economics, November 2006, 53 (8), 1857-1875.
- Schabert, Andreas, "Monetary policy under a fiscal theory of sovereign default", Journal of Economic Theory, 2010, 145 (2), 860-868.
- Gennaioli, Nicola, Alberto Martin, and Stefano Rossi, "Sovereign Default, Domestic Banks and Financial Institutions", SSRN eLibrary, 2010.

The general value function

$\mathbb{V}(y_t, z_{t-1} b_{t-1}, \vec{p}_t; \alpha) =$

$$\sup_{b_t \in B_t, \theta_t \in \Theta_t, g_t \in G_t, \tau_t \in \Upsilon_t} \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}(y_{t+1}, z_t b_t, \vec{p}_{t+1}(\theta_t); \alpha)\} \quad \text{s. t.}$$

$$c_t = y_t - x(\tau_t) - \tau_t + \alpha(1-\theta_t) z_{t-1} b_{t-1} - \alpha b_t$$

$$(1-\theta_t) z_{t-1} b_{t-1} + g_t = \tau_t + b_t - p(\theta_t) - \vec{p}_t$$

$$z_t = \inf[z : \ z_t E_t[1-\theta_{t+1}] = R_t]$$

The simplified value function

$\mathbb{V}(y_t, z_{t-1}b_{t-1}, \vec{p}_t; \alpha) =$

$$\sup_{b_t \in B_t, \theta_t \in [0,1], g_t \in G_t} \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}(z_t b_t, \vec{p}_{t+1}(\theta_t))\} \quad \text{s.t.}$$

$$\begin{aligned} c_t &= y_t - x(\tau_t) - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} \\ &\quad + (1-\alpha)b_t - g_t - p(\theta_t) - \vec{p}_t \\ \tau_t &= (1-\theta_t)z_{t-1}b_{t-1} + g_t - b_t + p(\theta_t) + \vec{p}_t \\ z_t &= \inf[z : \; z_t E_t[1-\theta_{t+1}] = R_t] \end{aligned}$$

Financial market exclusion

The value function with financial market access

$$\mathbb{V}^f = \max\left(\mathbb{V}^d, \mathbb{V}^n\right)$$

The value of non-default

$$\mathbb{V}^n(z_{t-1}b_{t-1}) = \sup_{b_t \in B_t, g_t \in G_t} \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}^f(z_t b_t)\} \quad \text{s. t.}$$

$$c_t = y_t - x(\tau_t) - (1-\alpha)z_{t-1}b_{t-1} + (1-\alpha)b_t - g_t$$

$$\tau_t = z_{t-1}b_{t-1} + g_t - b_t$$

$$z_t = \inf[z : \; z_t E_t[1-\theta_{t+1}] = R_t]$$

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$$z_t = \inf[z : \ z_t E_t[1-\theta_{t+1}] = R_t]$$

Financial market exclusion

The value of default $\mathbb{V}^d(z_{t-1}b_{t-1}) =$

$$\sup_{\theta_t \in [0,1], g_t \in G_t} \left\{ \mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t \mathbb{V}^f(0) + (1-\kappa) E_t \mathbb{V}^a(\vec{p}_{t+1}(\theta_t)) \right] \right\} \quad \text{s. t.}$$

$$c_t = y_t - x(\tau_t) - (1-\alpha)(1-\theta_t) z_{t-1} b_{t-1} - g_t - p(\theta_t)$$

$$\tau_t = (1-\theta_t) z_{t-1} b_{t-1} + g_t + p(\theta_t)$$

The value of anarchy $\mathbb{V}^a(\vec{p}) =$

$$\sup_{g_t \in G_t} \left\{ \mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t \mathbb{V}^f(0) + (1-\kappa) E_t \mathbb{V}^a(\vec{p}) \right] \right\} \quad \text{s. t.}$$

$$c_t = y_t - x(\tau_t) - g_t - \vec{p}$$

$$\tau_t = g_t + \vec{p}$$

Financial market exclusion

The value of default $\mathbb{V}^d(z_{t-1}b_{t-1}) =$

$$\sup_{\theta_t\in[0,1],g_t\in G_t} \{\mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t\mathbb{V}^f(0) + (1-\kappa)E_t\mathbb{V}^a(\vec{p}_{t+1}(\theta_t))\right]\} \quad \text{s. t.}$$

$$c_t = y_t - x(\tau_t) - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} - g_t - p(\theta_t)$$

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The value of anarchy $\mathbb{V}^a(\vec{p}) =$

$$\sup_{g_t\in G_t} \{\mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t\mathbb{V}^f(0) + (1-\kappa)E_t\mathbb{V}^a(\vec{p})\right]\} \quad \text{s. t.}$$

$$c_t = y_t - x(\tau_t) - g_t - \vec{p}$$

$$\tau_t = g_t + \vec{p}$$

The Eaton&Gersovitz type model

$\mathbb{V}(y_t, (1-\alpha)z_{t-1}b_{t-1}, \vec{p}_t) =$

$$\sup_{b_t \in B_t, \theta_t \in [0,1], g_t \in G_t} \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}(y_{t+1}, (1-\alpha)z_t b_t, \vec{p}_{t+1})\} \quad \text{s. t.}$$

$$c_t = y_t - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1}$$
$$+ (1-\alpha)b_t - g_t - p_t - \vec{p}_t$$
$$z_t = \inf[z : \; z_t E_t[1-\theta_{t+1}] = R_t]$$

The value of default

$\mathbb{V}^d(y_t, (1-\alpha)z_{t-1}b_{t-1}) =$

$$\sup_{\theta_t \in [0,1], g_t \in G_t} \left\{ \mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t \mathbb{V}^f(y_{t+1}, 0) + (1-\kappa) E_t \mathbb{V}^a_{t+1}(y_{t+1}, \vec{p}) \right] \right\}$$

$$c_t = y_t - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} - g_t - p_t$$

$\mathbb{V}^a_t(y_t, \vec{p}) =$

$$\sup_{g_t \in G_t} \left\{ \mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t \mathbb{V}^f(y_{t+1}, 0) + (1-\kappa) E_t \mathbb{V}^a_{t+1}(y_{t+1}, \vec{p}) \right] \right\} \quad \text{s. t.}$$

$$c_t = y_t - g_t - \vec{p}$$

The value of default

$\mathbb{V}^d(y_t, (1-\alpha)z_{t-1}b_{t-1}) =$

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$$c_t = y_t - g_t - \vec{p}$$

The optimal default rate

$\mathbb{V}^d(y_t, (1-\alpha)z_{t-1}b_{t-1}) =$

$$\sup_{\theta_t \in [0,1], g_t \in G_t} \{\mathbb{U}(c_t, g_t) + \beta\left[\kappa E_t \mathbb{V}^f(y_{t+1}, 0) + (1-\kappa)E_t \mathbb{V}^a_{t+1}(y_{t+1}, \vec{p})\right]\}$$

$$c_t = y_t - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} - g_t - p_t$$

$$\mathbb{V}^a_\tau(y_\tau, \vec{p}) = \mathbb{U}((y_\tau - g^a_\tau - \vec{p}), g^a_\tau) + \beta\left[\kappa E_\tau \mathbb{V}^f(y_{\tau+1}, 0) + (1-\kappa)E_\tau \mathbb{V}^a_{\tau+1}(y_{\tau+1}, \vec{p})\right]$$

FOC over θ_t:

$$\frac{\partial \mathbb{V}^d}{\partial \theta_t} = \mathbb{U}_1(c_t, g_t) * (1-\alpha)z_{t-1}b_{t-1} \Rightarrow$$

if $\alpha = 1$ then $\theta_t = 0$

otherwise $\theta_t = 1$ given default decision

The optimal default rate

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$$c_t = y_t - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} - g_t - p_t$$

$$\mathbb{V}^a_\tau(y_\tau, \vec{p}) = \mathbb{U}((y_\tau - g^a_\tau - \vec{p}), g^a_\tau) + \beta \left[\kappa E_\tau \mathbb{V}^f(y_{\tau+1}, 0) + (1-\kappa) E_\tau \mathbb{V}^a_{\tau+1}(y_{\tau+1}, \vec{p})\right]$$

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The value of full default

$\mathbb{V}^d(y_t, (1-\alpha)z_{t-1}b_{t-1}) =$

$$\sup_{g_t \in G_t} \left\{ \mathbb{U}(c_t, g_t) + \beta \left[\kappa E_t \mathbb{V}^f(y_{t+1}, 0) + (1-\kappa) E_t \mathbb{V}^a_{t+1}(y_{t+1}, \vec{p}) \right] \right\} \quad \text{s.}$$

$$c_t = y_t - g_t - p_t$$

$$\mathbb{V}^a_\tau(y_\tau, \vec{p}) = \mathbb{U}((y_\tau - g^a_\tau - \vec{p}), g^a_\tau) + \beta \left[\kappa E_\tau \mathbb{V}^f(y_{\tau+1}, 0) + (1-\kappa) E_\tau \mathbb{V}^a_{\tau+1}(y_{\tau+1}, \vec{p}) \right]$$

Conclusion

The value of full default is independent from α

The value of full default

$$\mathbb{V}^d(y_t, (1-\alpha)z_{t-1}b_{t-1}) =$$

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Conclusion

The value of full default is independent from α

The value of non-default

$$\mathbb{V}^n((1-\alpha)z_{t-1}b_{t-1}) =$$

$$\sup_{b_t \in B_t, g_t \in G_t} \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}^f((1-\alpha)z_t b_t)\} \quad \text{s.t.}$$
$$c_t = y_t - (1-\alpha)z_{t-1}b_{t-1} + (1-\alpha)b_t - g_t$$
$$z_t = \inf[z : \; z_t(1-\lambda_t) = R_t]$$

More compact:

$$\mathbb{V}^n((1-\alpha)z_{t-1}b_{t-1}) =$$

$$\mathbb{U}((y_t - (1-\alpha)z_{t-1}b_{t-1} + (1-\alpha)b_t^n - g_t^n), g_t^n)$$
$$+ \beta E_t \mathbb{V}^f((1-\alpha)z_t b_t^n) \quad \text{s.t.}$$
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$$z_t = \inf[z : \; z_t(1-\lambda_t) = R_t]$$

The value of non-default

Theorem (1)

For any $0 \leq \alpha_1 < \alpha_2 < 1$ *and any given outstanding amount of debt* $z_{t-1}b_{t-1} \geq 0$*, we have that*
$\mathbb{V}^n((1-\alpha_1)z_{t-1}b_{t-1}) \leq \mathbb{V}^n((1-\alpha_2)z_{t-1}b_{t-1})$.

Go to Proof 1

Conclusion

Compare $\mathbb{V}^d$ with $\mathbb{V}^n$ we conclude that the default probability $\lambda_t \equiv \Pr(\mathbb{V}^d_{t+1} > \mathbb{V}^n_{t+1})$ decreases with increasing α.

Skip Proof 1

The value of non-default

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Compare $\mathbb{V}^d$ with $\mathbb{V}^n$ we conclude that the default probability $\lambda_t \equiv \Pr(\mathbb{V}^d_{t+1} > \mathbb{V}^n_{t+1})$ decreases with increasing α.

Skip Proof 1

The proof

1.

$$
\begin{aligned}
& \mathbb{V}^n((1-\alpha_2)z_{t-1}b_{t-1}) \\
=& \mathbb{U}((y_t-(1-\alpha_2)z_{t-1}b_{t-1}+(1-\alpha_2)b^n_{2t}-g^n_{2t}),g^n_{2t}) \\
& \quad +\beta E_t\mathbb{V}^f((1-\alpha_2)z_{2t}b^n_{2t}) \\
\geq& \mathbb{U}((y_t-(1-\alpha_2)z_{t-1}b_{t-1}+(1-\alpha_2)\frac{1-\alpha_1}{1-\alpha_2}b^n_{1t}-g^n_{1t}),g^n_{1t}) \\
& \quad +\beta E_t\mathbb{V}^f((1-\alpha_2)z\left((1-\alpha_2)\frac{1-\alpha_1}{1-\alpha_2}b^n_{1t}\right)\frac{1-\alpha_1}{1-\alpha_2}b^n_{1t}) \\
\geq& U(y_t-(1-\alpha_1)z_{t-1}b_{t-1}+(1-\alpha_1)b^n_{1t}-g^n_{1t}),g^n_{1t}) \\
& \quad +\beta E_t\mathbb{V}^f((1-\alpha_1)z_{1t}b^n_{1t}) \\
=& \mathbb{V}^n((1-\alpha_1)z_{t-1}b_{t-1})
\end{aligned}
$$

$\square$

Back to theorem 1

The Calvo type model

the two-period case:

$$\mathbb{V}(z_0 b_0) = \sup_{\theta_1 \in [0,1], g_1 \in G_1} \{\mathbb{U}(c_1, g_1)\} \quad \text{s. t.}$$

$$c_1 = y_1 - x(\tau_1) - (1-\alpha)(1-\theta_1) z_0 b_0 - g_1 - p(\theta_1)$$

$$\tau_1 = (1-\theta_1) z_0 b_0 + g_1 + p(\theta_1)$$

and

$$z_0 = \inf[z : \; z_0 E_0[1-\theta_1] = R_0]$$

FOC over θ_1:

$$\mathbb{U}_1(c_1, g_1) * (x_1(\tau_1)(z_0 b_0 - p_1(\theta_1)) + (1-\alpha) z_0 b_0 - p_1(\theta_1)) = 0$$

The Calvo type model

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$$\mathbb{U}_1(c_1, g_1) * (x_1(\tau_1)(z_0 b_0 - p_1(\theta_1)) + (1-\alpha) z_0 b_0 - p_1(\theta_1)) = 0$$

The two-period case

Theorem (2)

In a two-period model, given $\mathbb{U}_1(c, g) > 0$, $x'(\tau^m) = 0$ with $\tau^m < \infty$ denoting the deadweight loss minimizing tax rate, $x''(\tau) > 0$ and $p''(\theta) \geq 0$, then $\frac{d\theta^*}{d\alpha} \leq 0$.

2.

From the FOC we have:

$$f \equiv x'(\tau_1)(z_0 b_0 - p'(\theta_1)) + (1 - \alpha) z_0 b_0 - p'(\theta_1) = 0$$

$$\frac{\partial f}{\partial \theta} = -x''(\tau_1)(z_0 b_0 - p'(\theta_1))^2 - x'(\tau_1) p''(\theta_1) - p''(\theta_1) < 0$$

$$\frac{\partial f}{\partial \alpha} = -z_0 b_0 < 0 \Rightarrow$$

$$\frac{d\theta}{d\alpha} = -\frac{\partial f / \partial \alpha}{\partial f / \partial \theta} < 0$$

$\square$

The two-period case

Theorem (2)

In a two-period model, given $\mathbb{U}_1(c, g) > 0$, $x'(\tau^m) = 0$ with $\tau^m < \infty$ denoting the deadweight loss minimizing tax rate, $x''(\tau) > 0$ and $p''(\theta) \geq 0$, then $\frac{d\theta^*}{d\alpha} \leq 0$.

2.

From the FOC we have:

$$f \equiv x'(\tau_1)(z_0 b_0 - p'(\theta_1)) + (1-\alpha)z_0 b_0 - p'(\theta_1) = 0$$

$$\frac{\partial f}{\partial \theta} = -x''(\tau_1)(z_0 b_0 - p'(\theta_1))^2 - x'(\tau_1)p''(\theta_1) - p''(\theta_1) < 0$$

$$\frac{\partial f}{\partial \alpha} = -z_0 b_0 < 0 \Rightarrow$$

$$\frac{d\theta}{d\alpha} = -\frac{\partial f/\partial \alpha}{\partial f/\partial \theta} < 0$$

$\square$

The infinite-horizon case

Theorem (3)

In an infinite-horizon model, given $\mathbb{U}_1(c,g) > 0$, $x'(\tau^m) = 0$ with $\tau^m < \infty$ denoting the deadweight loss minimizing tax rate, $x''(\tau) > 0$ and $p''(\theta) \geq 0$, then $\frac{d\theta^*}{d\alpha} \leq 0$.

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Consider the value function for the infinite-horizon model:

$$\mathbb{V}(y_t, z_{t-1}b_{t-1};\alpha) = \sup_{b_t \in B_t, \theta_t \in [0,1], g_t \in G_t} \{\mathbb{U}(c_t, g_t) + \beta E_t \mathbb{V}(y_{t+1}, z_t b_t; \alpha)\} \quad \text{s. t.}$$

$$c_t = y_t - x(\tau_t) - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} + (1-\alpha)b_t - g_t - p(\theta_t)$$

$$\tau_t = (1-\theta_t)z_{t-1}b_{t-1} + g_t - b_t + p(\theta_t)$$

$$z_t = \inf[z: \ z_t E_t[1-\theta_{t+1}] = R_t]$$

FOC over θ_1:

$$\mathbb{U}_1(c_t, g_t) * (x_1(\tau_t)(z_{t-1}b_{t-1} - p_1(\theta_t)) + (1-\alpha)z_{t-1}b_{t-1} - p_1(\theta_t)) = 0$$

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Good equilibrium and bad equilibrium

	Good equilibrium	**Bad equilibrium**
The contracted interest rate is		
	lower	higher
With higher outstanding amount of debt the interest rate		
	rises	falls
With a larger α value the interest rate		
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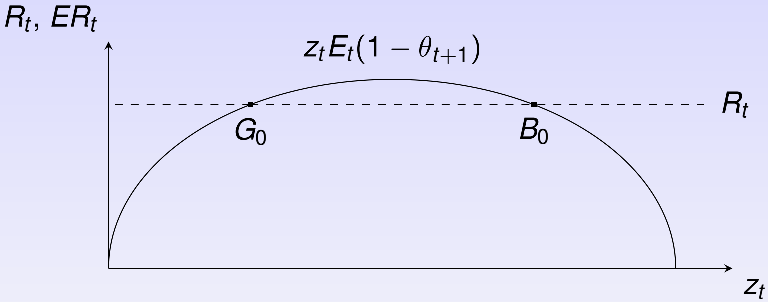

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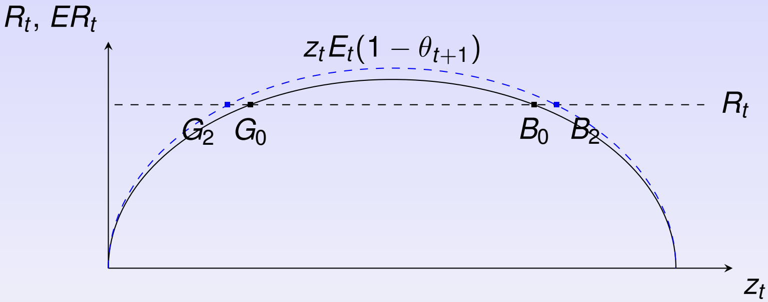

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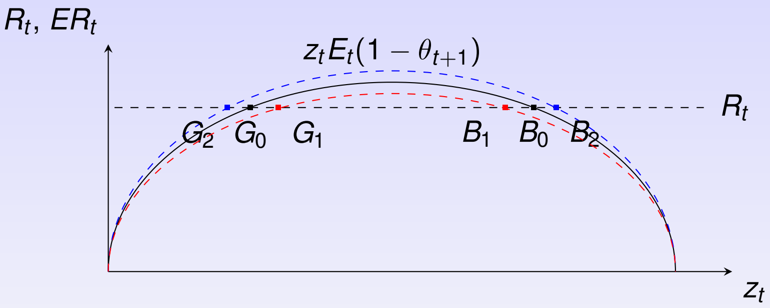

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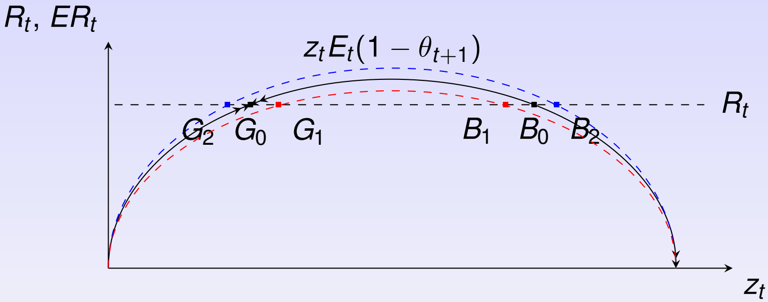

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The value of α is positively related to the capital availability in the domestic economy and in the economies with tight economic and/or political links to it if the government bond can be traded freely among the investors.

The extended Calvo Model

Example (Calvo extended)

$$\mathbb{V}(zb) = \sup_{\theta\in[0,1],g\in\{\bar{g}\},\tau\in\mathbb{R}} \{\mathbb{U}\left(y-\tau-x(\tau)+\alpha(1-\theta)zb, g\right)\}$$

$$\equiv \sup_{\theta\in[0,1],\tau\in\mathbb{R}} \{y-\tau-x(\tau)+\alpha(1-\theta)zb\} \quad \text{s. t.}$$

$$(1-\theta)zb+g=\tau-p(\theta)=\tau-\omega\theta zb$$

$$z=\inf[z:(1-\theta)z=R]$$

The extended Calvo Model

The deadweight loss:

$$x(0) = x'(0) = 0$$

$$x''(\tau) > 0 \quad \forall \tau$$

$$\lim_{\tau \to \infty} x''(\tau) = \infty = -\lim_{\tau \to -\infty} x''(\tau)$$

From budget constraint:

$$\theta = \frac{zb + g - \tau}{(1-\omega)zb}$$

Insert θ:

$$\inf_{\tau \in \mathbb{R}} \{x(\tau) - \frac{\omega - (1-\alpha)}{1-\omega}\tau\} \quad \Rightarrow \quad x(\tau*) = \frac{\omega - (1-\alpha)}{1-\omega}$$

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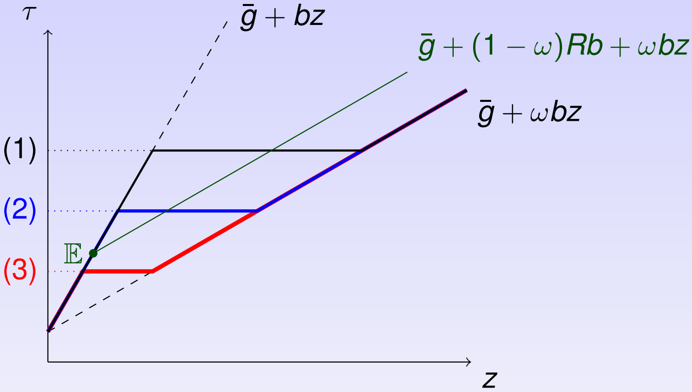
The equilibria under different α

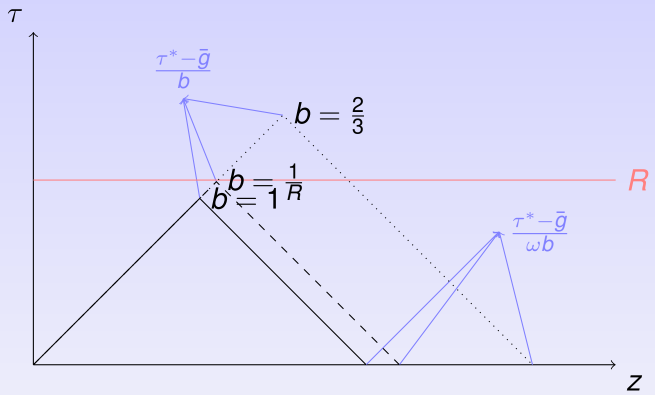


(1) $\tau_1^*(\alpha_1 = 1) = \bar{g} + Rb_1^{max}$

(2) $\tau_2^*(\alpha_2 < 1) = \bar{g} + Rb_2^{max}$

(3) $\tau_3^*(\alpha_3 \ll 1) = \bar{g} + Rb_3^{max}$

The equilibria under different b



The welfare effect

Assume $R' > R$:

$$\begin{aligned} c &= y - \tau - x(\tau) + \alpha Rb \\ &= y + R'b - g - (1-\alpha)Rb - x(g + Rb - R'b) \end{aligned}$$

FOC:

$$\frac{\partial c}{\partial b} = R' - (1-\alpha)R + x'(g + Rb - R'b)(R' - R) > 0$$

Domestic tax payers only:

Also $c^d = y + R'b - g - Rb - x(g + Rb - R'b)$ strictly increases with rising b resulting from a higher α.

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The extended Eaton&Gersovitz Model

Example (Eaton&Gersovitz extended)

$$c_t = y_t - (1-\alpha)(1-\theta_t)z_{t-1}b_{t-1} + (1-\alpha)b_t$$

$$\max_{\theta_t \in [0,1]} E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_\tau - P_\tau) \right]$$

$$z_t(1-\lambda_t) = R_t$$

The value functions

The value of default:

$$\mathbb{V}_t^D = E_t\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(y_\tau - P_\tau)\right]$$

The value of non-default

$$\mathbb{V}_t^R((1-\alpha)z_{t-1}b_{t-1})$$

$$= \sup_{b_t} U(y_t - (1-\alpha)z_{t-1}b_{t-1} + (1-\alpha)b_t) + \beta E_t[\mathbb{V}_{t+1}^f((1-\alpha)z_t b_t)]$$

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Proof.

Let $0 \leq \alpha_1 < \alpha_2 < 1$:

$$
\begin{aligned}
&\mathbb{V}_t^R((1-\alpha_1)z_{t-1}b_{t-1}) \\
=&U(y_t - (1-\alpha_1)z_{t-1}b_{t-1} + (1-\alpha_1)b_{1t}^*) + \beta E_t[\mathbb{V}_{t+1}((1-\alpha_1)z_{1t}^* b_{1t}^*)] \\
\geq&U(y_t - (1-\alpha_1)z_{t-1}b_{t-1} + (1-\alpha_1)\frac{1-\alpha_2}{1-\alpha_1}b_{2t}^*) + \beta E_t[\mathbb{V}_{t+1}((1-\alpha_1)z_{2t}^*\frac{1-\alpha_2}{1-\alpha_1}b_{2t}^*)] \\
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λ_t is decreasing with rising α.

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=& \mathbb{V}_t^R((1-\alpha_2)z_{t-1}b_{t-1})
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- equivalently, α can be viewed as the congruence between the wealth position of the ultimate lenders and the domestic private wealth or the aggregate wealth of the voters.
- α lies in the interval $[0, 1]$ and its exact value is positively related to the capital availability in the borrowing economy and in the economies economically or politically closely linked to it.