Constrained Poisson Pseudo Maximum Likelihood Estimation of Structural Gravity Models

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Motivation

- Dummy PPML treats trade resistances as parameters of exporter and importer dummies. Since trade resistances change in counterfactual experiments, dummy PPML does not provide standard errors and confidence intervals of counterfactual predictions.
- Assume that the system of trade resistances is solved at true parameters in expectation at given observed output and expenditure shares (or GDP-shares) as in Anderson and van Wincoop (2003).

  Then trade resistances are fully determined by the structural parameters suggesting a constrained estimation approach.
- The dummy PPML as proposed by Santos Silva and Tenreyro (2006) is consistent in this setting. But Monte Carlo results indicate that it overrejects t-tests on the structural parameters under this assumption.

The contribution

- Provide a constrained projection based estimation procedure based on Heyde and Morton (1993) and Falocci, Paniccià and Stanghellini (2009).
- Derivation the asymptotic distribution of the constrained PPML estimator under the assumption that the number of restrictions grows with sample size.
- Confidence intervals for comparative static predictions, which are unavailable under dummy PPML.
- Check the performance in a Monte-Carlo analysis.
- Apply the constrained PPML to estimate the impact of a common official language, contiguity and country borders on international trade flows and welfare quantitatively.

The structural gravity model of AvW (2003) I

Consider bilateral trade flows between exporter countries $i = 1, ..., C$ and importer counties $j = 1, ..., C$ (Anderson and van Wincoop, 2003):

$$\begin{aligned} x_{ij,C} &= Y_W t_{ij}^{1-\sigma} \theta_{j,C} P_{j,C}^{\sigma-1} \theta_{i,C} \Pi_{i,C}^{\sigma-1} \eta_{ij} \\ &= Y_W e^{z'_{ij}\alpha + \beta_{i,C}(\alpha) + \gamma_{j,C}(\alpha)} + \varepsilon_{ij} \\ &= Y_W m_{ij}(\alpha) + \varepsilon_{ij} \end{aligned}$$

$$\begin{aligned} t_{ij}^{1-\sigma} &= e^{z'_{ij}\alpha} \\ e^{\beta_{i,C}(\alpha)} &= \theta_{i,C} \Pi_i(\alpha)^{\sigma-1} \\ e^{\gamma_{j,C}(\alpha)} &= \theta_{j,C} P_j(\alpha)^{\sigma-1} \end{aligned}$$

The structural gravity model of AvW (2003) II

The system of trade resistances:

$$
\begin{aligned}
\theta_{i,C} &= \sum_{j=1}^{C} e^{z_{ij}'\alpha + \beta_{i,C}(\alpha) + \gamma_{j,C}(\alpha)} \\
\theta_{j,C} &= \sum_{i=1}^{C} e^{z_{ij}'\alpha + \beta_{i,C}(\alpha) + \gamma_{j,C}(\alpha)} \\
0 &= D' m(\vartheta_C) - \theta_C
\end{aligned}
$$

The score of the PPML I

Treating the trade resistance terms as dummies and maximizing the Poisson log-likelihood under the restriction $D'm(\vartheta_C) - \theta_C$ for a given selection matrix V leads to the score

$$
\begin{aligned}
\frac{\partial \ln L^C(\vartheta_C|V)}{\partial \alpha} &= Z'V(s_C - m(\vartheta_C)) + Z'M(\vartheta_C)D\lambda \\
\frac{\partial \ln L^C(\vartheta_C|V)}{\partial (\beta_C', \gamma_C')'} &= D'V(s_C - m(\vartheta_C)) + D'M(\vartheta_C)D\lambda \\
\frac{\partial \ln L^C(\vartheta_C|V)}{\partial \lambda} &= D'm(\vartheta_C) - \theta_C = 0.
\end{aligned}
$$

The score of the PPML II

The dummy PPML ignores the non-stochastic nature of this restriction.

Assume no missings and $V = I$.

The DGP satisfies $D'm(\vartheta_{0,C}) = \theta_C$.

Dummy PPML sets the score $D'(s_C - m(\vartheta_C))$ to zero.

But $D'(m(\vartheta_{0,C}) + \varepsilon) - m(\vartheta_{0,C})) = D'\varepsilon$!

So the system of trade resistances does not hold under true structural parameters.

Proposition 1 (Constrained PPML). *Assume iteration r yields $\widehat{\vartheta}_{C,r}$ and*

$$
\begin{aligned}
\widehat{h}_r &= \theta_C - D' m(\widehat{\vartheta}_{C,r}) + D' M(\widehat{\vartheta}_{C,r}) W \widehat{\vartheta}_{C,r} \\
\widehat{F}_r &= D' M(\widehat{\vartheta}_{C,r}) W, \\
\widehat{G}_r &= W' V M(\widehat{\vartheta}_{C,r}) V W, \\
W &= [Z, D] \\
\widehat{\vartheta}_{C,r+1} - \widehat{\vartheta}_{C,r} &= \left(\widehat{G}_r^{-1} - \widehat{G}_r^{-1} \widehat{F}_r' \left(\widehat{F}_r \widehat{G}_r^{-1} \widehat{F}_r' \right)^{-1} \widehat{F}_r \widehat{G}_r^{-1} \right) \\
&\quad * W' V \left(s_C - m(\widehat{\vartheta}_{C,r}) \right) \\
&\quad + \widehat{G}_r^{-1} \widehat{F}_r' \left(\widehat{F}_r \widehat{G}_r^{-1} \widehat{F}_r' \right)^{-1} \left(\widehat{h}_r - \widehat{F}_r \widehat{\vartheta}_{C,r} \right)
\end{aligned}
$$

Upon convergence, when $\widehat{\vartheta}_{C,r+1} = \widehat{\vartheta}_{C,r}$ it holds that

$$
\begin{aligned}
0 &= \widehat{h}_r - \widehat{F}_r \widehat{\vartheta}_{C,r} = \theta_C - D' m(\widehat{\vartheta}_{C,r}) \\
0 &= \left(\widehat{G}_r^{-1} - \widehat{G}_r^{-1} \widehat{F}_r' \left(\widehat{F}_r \widehat{G}_r^{-1} \widehat{F}_r' \right)^{-1} \widehat{F}_r \widehat{G}_r^{-1} \right) \\
&\quad * \left(W' V \left(s_C - m(\widehat{\vartheta}_{C,r}) \right) \right).
\end{aligned}
$$

Proposition 2 (Asymptotic distribution of $\hat{\alpha}$). *Under a set of assumptions specified in the paper for the iterated PPML it holds that*

(i) $\widehat{\alpha} \xrightarrow{p} \alpha_0$, *constrained PPML is consistent.*

(ii) $C(\hat{\alpha} - \alpha_0) \xrightarrow{d} N\left(0, B_0^{-1} A_0 \Omega_\varepsilon A_0' B_0^{-1}\right)$,

where $\Omega_\varepsilon = diag(\sigma_{ij}^2)$, $A_0 \Omega_\varepsilon A_0' = p\lim_{C\to\infty} \frac{1}{C^2} A(\alpha^*)\varepsilon\varepsilon' A(\alpha^*)'$,
$B_0 = \lim_{C\to\infty} B(\alpha^*)$

$$
\begin{aligned}
A(\alpha) &= C^2 \left[I_K, 0_{K\times 2C-1}\right] \\
&\quad * \left(I - F(\alpha)' \left(F(\alpha)G(\alpha)^{-1}F(\alpha)'\right)^{-1} F(\alpha)G(\alpha)^{-1}\right) \\
&\quad * W'V\varepsilon \\
B(\alpha) &= Z'V\left[M(\alpha) - M(\alpha)D(D'M(\alpha)D)^{-1}D'M(\alpha)\right]Z.
\end{aligned}
$$

(iii) $\widehat{B} = B(\widehat{\alpha}) \xrightarrow{p} B_0$ *and* $\frac{1}{C^2} A(\widehat{\alpha}) diag(\widehat{\varepsilon}\widehat{\varepsilon}') A(\widehat{\alpha})' \xrightarrow{p} A_0 \Omega_\varepsilon A_0'$.

If trade flows are fully observed and data are generated such that $D' s_C = \theta_C$, as in GTAP or WIOD:

$$D' \varepsilon = 0.$$

Under fully observed trade flows dummy and constrained PPML coincide.

By construction the disturbances will not be independent in this case. Then one can decompose the restrictions as

$$\begin{aligned} D' \varepsilon &= D'_R \varepsilon_R + D'_U \varepsilon_U = 0 \\ \varepsilon_R &= -D'^{-1}_R D'_U \varepsilon_U. \end{aligned}$$

Apply Proposition 2 with

$$\begin{aligned} A(\alpha) &= C^2 \left[-Z_R D'^{-1}_R D'_U + Z'_U \right] \\ \Omega_U &= E[\varepsilon_U \varepsilon'_U]. \end{aligned}$$

Proposition 3 (Counterfactual prediction). *Define* $V_\alpha = B_0^{-1} A_0 \Omega_\varepsilon A_0 B_0^{-1}$ *and the normalized selection matrix* S *so that* $SM(\alpha_0, Z^c)$ *possesses typical non-zero element* $C^2 m_{ij}(\alpha_{0,} z_{ij}^c)$. *Let*

$$\Gamma_0^c = \lim_{C\to\infty} SM(\alpha_0, Z^c)[I - D\left(D'M(\alpha_0, Z^c)D\right)^{-1} D'M(\alpha_0, Z^c)]Z^c$$

Then, it follows that

$$CS(m(\widehat{\alpha}, Z) - m(\alpha_0, Z)) \xrightarrow{d} N(0, \Gamma_0 V_\alpha \Gamma_0')$$

and $\widehat{\Gamma}^c \widehat{V}_\alpha \widehat{\Gamma}^c - \Gamma_0 V_\alpha \Gamma_0' = o_p(1)$.

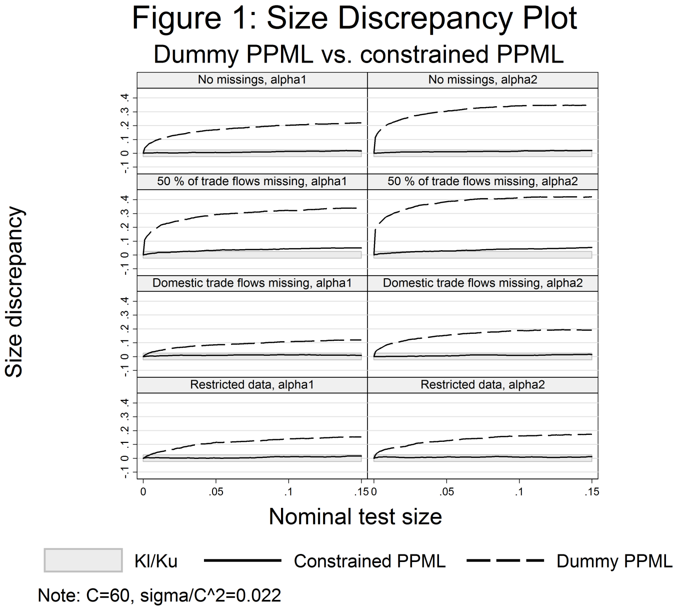
As similar approach allows to derive confidence intervals for percentage changes.

Table 1: Simulated standard errors, estimated standard errors and 95% coverage rates of structuralarameters under constrained PPML

Countries	Missings	Simulated std.		Estimated std.		95% Cov. rate	
		α_1	α_2	α_1	α_2	α_1	α_2
40	0	0.0117	0.0030	0.0110	0.0028	0.938	0.939
40	50%	0.0147	0.0043	0.0136	0.0040	0.924	0.919
40	domestic	0.0089	0.0019	0.0087	0.0018	0.939	0.944
60	0	0.0048	0.0013	0.0046	0.0012	0.944	0.940
60	50%	0.0078	0.0020	0.0070	0.0018	0.924	0.929
60	domestic	0.0037	0.0008	0.0036	0.0008	0.941	0.945

Notes: 5000 Monte Carlo runs. Coverage rate refers to a nominal 95 percent confidence interval using the normal distribution.

Monte Carlo Simulation II, t-tests



Monte Carlo Simulation III

Table 2: 95% coverage rates counterfactual changes under constrained PPML, relative change

Countries	Missings	Country pairs			
		1,1	1,2	2,1	2,2
40	none	0.950	0.937	0.948	0.951
40	50%	0.939	0.922	0.936	0.939
40	domestic	0.952	0.938	0.948	0.953
60	none	0.950	0.954	0.951	0.957
60	50%	0.933	0.937	0.933	0.941
60	domestic	0.943	0.951	0.948	0.949

Notes: 5000 Monte Carlo runs. The coverage rate refers to a nominal 95 percent confidence interval using the normal distribution.

Monte Carlo Simulation IV

Table 3: 95% coverage rates of counterfactual changes with restricted observed domestic trade flows,

Countries	Country pairs			
	1,1	1,2	2,1	2,2
Absolute Change				
40	0.954	0.945	0.941	0.936
60	0.928	0.946	0.946	0.926
Relative Change				
40	0.950	0.946	0.947	0.952
60	0.954	0.950	0.951	0.954

Notes: 5000 Monte Carlo runs. The coverage rate refers to a nominal 95 percent confidence interval using the normal distribution.

Estimation results I

Table 4: Parameter estimates, dummy and constrained PPML without domestic trade flows

	Dummy PPML		constrained PPML	
	α	t-value	α	t-value
County border	-1.42	-4.97^{***}	-1.42	-4.60^{***}
Common official language	-0.33	-3.46^{***}	-0.33	-2.58^{***}
Contiguity	-0.43	-4.28^{***}	-0.43	-2.76^{***}
Colony	0.11	1.11	0.11	0.58
Common Colonizer	-0.01	-0.02	-0.01	-0.03
Log distance	-0.91	-20.25^{***}	-0.91	-17.77^{***}
Pseudo-R^2	0.997		0.997	

Notes: The estimates are based on 3374 observations. Pseudo-R^2 is defined as the correlation of observed an predicted values.

Estimation results II

Table 5: Border effects

	Direct change in %			Total change in %		
	Impact	[95% CoI]		Impact	[95% CI]	
Dom-small	0.00	-	-	-69.82	-85.39	-54.26
Dom-large	0.00	-	-	-36.84	-56.16	-17.52
Small-small	75.74	61.11	90.37	3.84	-6.92	14.60
Large-large	75.74	61.11	90.37	91.77	66.69	116.86
Small-large	75.74	61.11	90.37	44.45	32.95	55.95
Large-small	75.74	61.11	90.37	38.73	31.68	45.79

Notes: ** significant at 5 %, *** significant at 1 %.