Monotone Comparative Statics for the Industry Composition

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Overview

- We let heterogeneous firms face decisions on a number of complementary activities in a monopolistically-competitive industry.
- How do the equilibrium optimal decisions respond to exogenous changes in the parameters of firms' profit maximisation problem?
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- Monotone comparative statics at the firm level is neither necessary nor sufficient for monotone comparative statics for the industry composition.
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    - The former is defined as all firms increasing their levels of the activities.
    - The latter is defined as FSD shifts in the equilibrium distributions of activity levels (implies that the mean increases).
- We provide sufficient conditions for MCS at the firm and industry levels and compare.
    - Possibility: strong and clear interdependence of activities at the industry level while the firm level is a mess.

Example: Melitz (2003)

- Exporting and labour input are complementary to each other and to lower variable trade costs. Firms differ w.r.t. productivity.
- In equilibrium, reductions in trade costs lead to fiercer competition.
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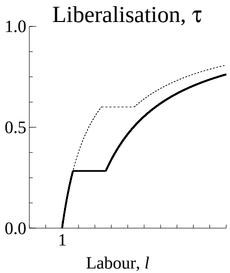
- In equilibrium, reductions in trade costs lead to fiercer competition.
- Following a reduction in variable trade costs, ex-post nonexporters reduce their use of labour while exporters increase their use of labour.
- When productivity is Pareto distributed, the result of the reduction in the variable trade cost on the firm-size distribution is a FSD shift. Moreover, the fraction of exporters increases.



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- Common functional forms (e.g. CES, Pareto, and others) result in industry-level monotone comparative statics which give rise to new and testable predictions.
- We show how these predictions survive under weaker assumptions (e.g. on the productivity distribution).

Related Literature

- Mrazova and Neary (2013).
- Costinot (2007,2009).
- Topkis (1978) and Milgrom and Shannon (1994).
- Bache and Laugesen (2014); see Application II below.
- Antràs, Fort, and Tintelnot (2014); see Application III below.
- Head, Mayer, and Thoenig (2014).
- Zhelobodko, Kokovin, Parenti, and Thisse (2012).
- Feenstra (2014).

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$$x^*(A\theta, \beta) = \arg\max_x \pi(x, A\theta, \beta)$$

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- Upon entry, firms learn their productivity level which is a realisation of a random variable with c.d.f. F.
- Free entry implies that expected profits of prospective entrants, Π, equal the fixed cost of entry, f_e, which determines $A = A(\beta)$.

$$f_e = \Pi(A, \beta) \left(= \int_{\theta \in \Theta} \max\{0, \pi^*(A\theta, \beta)\} \, \mathrm{d}F(\theta) \right)$$

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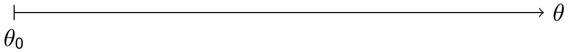
- In general, the demand level can therefore either rise or fall.
- Consider the effect on the equilibrium optimal decision, $\tilde{x}^*(\theta, \beta) = x^*(A(\beta)\theta, \beta)$:

$$\frac{\mathrm{d}\tilde{x}^*}{\mathrm{d}\beta} = \underbrace{\frac{\partial x^*}{\partial \beta}}_{\text{direct effect } (+)} + \underbrace{\frac{\partial x^*}{\partial A}\frac{\mathrm{d}A}{\mathrm{d}\beta}}_{\text{indirect effect } (?)}$$

- The equilibrium optimal decision of individual firms is unambiguously increasing only when the direct and indirect effects are aligned (A increases)!
- In many cases, such as trade liberalisations, A is however falling.

Industry Composition



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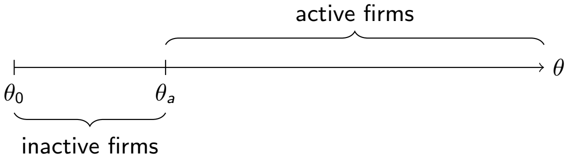

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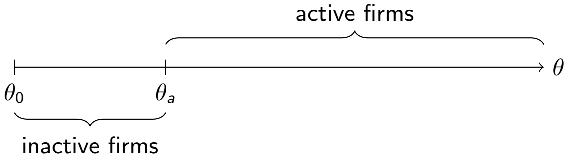

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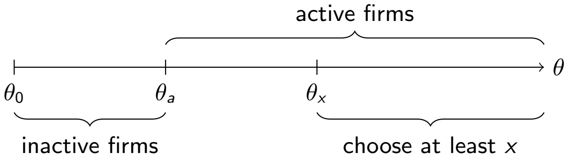

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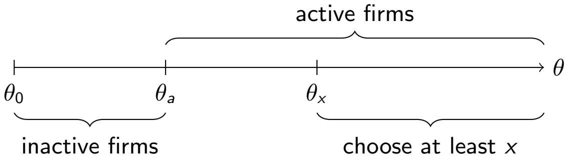

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- The share of firms choosing at least x, $s_x(x, A, \beta) = 1 - F(\theta_x)$.

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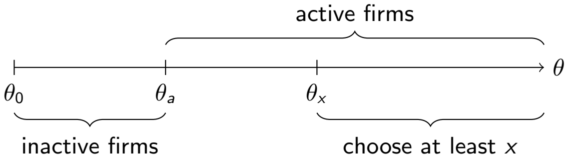

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- The share of firms choosing at least x, $s_x(x, A, \beta) = 1 - F(\theta_x)$.
- Industry composition represented by c.d.f of equilibrium values of x,

$$H(x, \beta) = 1 - \frac{s_x(x, A, \beta)}{s_a(A, \beta)}.$$

Industry-Level Comparative Statics I

- Consider the effect of β on H (the industry composition),

$$\frac{1}{1-H}\frac{\mathrm{d}H}{\mathrm{d}\beta} = \underbrace{-\frac{1}{s_x}\frac{\mathrm{d}s_x}{\mathrm{d}\beta}}_{\text{level effect}} + \underbrace{\frac{1}{s_a}\frac{\mathrm{d}s_a}{\mathrm{d}\beta}}_{\text{selection effect}}$$

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- The selection effect introduces a discrepancy between firm- and industry-level comparative statics.
- Expanding and rearranging gives

$$\frac{1}{1-H}\frac{\mathrm{d}H}{\mathrm{d}\beta} = \underbrace{-\frac{1}{s_x}\frac{\partial s_x}{\partial \beta}}_{\text{DLE}\leq 0} + \underbrace{\frac{1}{s_a}\frac{\partial s_a}{\partial \beta}}_{\text{DSE}\leq 0} \underbrace{-\left(\frac{1}{s_x}\frac{\partial s_x}{\partial A} - \frac{1}{s_a}\frac{\partial s_a}{\partial A}\right)\frac{\mathrm{d}A}{\mathrm{d}\beta}}_{\text{IE}}$$

- The direct level effect is always nonpositive.
- The direct selection effect is assumed nonpositive.

Industry-Level Comparative Statics II

- Question: When is

$$\text{Indirect Effects} = -\left(\frac{1}{s_x}\frac{\partial s_x}{\partial A} - \frac{1}{s_a}\frac{\partial s_a}{\partial A}\right)\frac{\mathrm{d}A}{\mathrm{d}\beta}$$

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  - If $\log\theta$ is distributed with decreasing hazard rate, then $\frac{1}{s_x}\frac{\partial s_x}{\partial A} < \frac{1}{s_a}\frac{\partial s_a}{\partial A}$.
    $\Rightarrow$ FSD shift in H if A falls.

Firm-Level Versus Industry-Level Comparative Statics

- Depending on the hazard rate of log-productivity, the firm- and industry-level comparative statics may or may not correspond.

Monotone Comparative Statics?

	Firm Level	Industry Level		
		IHLP	DHLP	CHLP
$A\uparrow$	$+$	$+$	$-$	$+$
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- In the knife-edge case of constant hazard rate of log-productivity, the industry level exhibits monotone comparative statics regardless of what happens at the firm level.
  - $\log\theta$ having constant hazard rate corresponds to productivity, θ, being Pareto distributed.

Generalisation

- First off, differentiability (plus concavity and continuity) is not important (lattice theory and supermodularity).
- Can also do comparative statics with respect to the choice set of firms.
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  - Multidimensional parameters, β.
  - Multidimensional decisions, x.
  - Multidimensional firm heterogeneity through additional characteristics, γ.
- Last point is important for generating plausible sorting patterns of firms into decisions.
  - The equilibrium decision is increasing in productivity which may be too strict of a relationship in the absence of other sources of heterogeneity – especially with multidimensional decisions.

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- A lot of heterogeneous-firms trade models comply with the (extended) setup above. Complementarities arise naturally in these and productivity being Pareto distributed is standard.
  - Describe the mechanics of industry-level results.
  - Can easily derive novel predictions from existing models.
  - Makes it relatively easy to see what arises from extending or combining these models.
- Second, we provide a general framework for modelling clear shifts in the industry composition while allowing for ambiguities at the firm level.
  - Observed shifts in IC may be due to more attractive complementary activities.

Application I: Trade Liberalisation and Firm-Size

- The prediction of a FSD shift in the firm-size distribution following trade liberalisations is valid for a number of extensions of Melitz (2003) which all feature complementarities and use the Pareto:
  - Bustos (2011): technology upgrading
  - Helpman, Melitz and Yeaple (2004): horisontal FDI
  - Arkolakis (2010): marketing expenditure
  - Helpman and Itskhoki (2010), Helpman, Itskhoki, and Redding (2010), Davis and Harrigan (2011), and Egger and Koch (2013) if one considers the expenditure on labour input.
- This prediction has not been emphasised since labour input is often maximised out in the profit functions presented in these papers (their focus is elsewhere).

Application II: Trade Liberalisation and Vertical Integration

- Bache and Laugesen (2014) combine the Melitz (2003) model of exporting with the Antràs and Helpman (2004) model of vertical integration and offshoring of intermediate-input production.
- **Slight** simplification unveils a natural complementarity between the three activities.
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- The consequences for sorting are remedied by letting firms be heterogeneous with respect to headquarter (factor) intensity as well as productivity.
- In our framework, liberalisation of final-goods or intermediate-goods trade leads to a higher prevalence of vertical integration.
    - Antràs and Helpman (2004) find the opposite.
    - Firm-level responses are ambiguous.

Application III: Antràs, Fort, and Tintelnot (2014)

- AFT express the problem of determining the optimal exporting and importing strategies of a firm from country l with productivity θ as

$$\max_{I_{lj}^M \in \{0,1\}_{j=1}^2,\ I_{lk}^X \in \{0,1\}_{k=1}^2} \pi_l = \left(\gamma \sum_{j=1}^2 I_{lj}^M T_j (\tau_{lj} w_j)^{-\psi}\right)^{(\sigma-1)/\psi} \sum_{k=1}^2 I_{lk}^X (\tau_{lk}^X)^{1-\sigma} A_k \theta$$

$$- w_l \sum_{j=1}^2 I_{lj}^M f_{lj} - w_l \sum_{k=1}^2 I_{lk}^X f_{lk}^X .$$

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- Firms make extensive margin import and export decisions. Assume that $(\sigma - 1)/\psi > 1$.
- Assume that the least productive active firms do neither export final goods nor import intermediate inputs. Wages are determined through an outside good.
- Under free entry, trade liberalisation (an increase in β_l) implies that the fractions of importers and exporters increase in country l if e.g. productivity θ is distributed Pareto while A_l is nonincreasing and A_j is nondecreasing, $j \neq l$.
- Under the same circumstances, the fractions of importers and exporters decrease in country, $j \neq l$.