

The implementation of monetary and fiscal rules in the EMU: a welfare-based analysis*

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Abstract

This paper implements a methodology to evaluate the desirability of monetary and fiscal rules within the context of the EMU using a DSGE model within a New Keynesian framework with sticky prices. The approach adopted is a welfare-based criterion that measures the welfare losses associated with these rules through a welfare loss function. Monetary policy follows a standard Taylor rule augmented by a stochastic component, driven by a union-wide monetary shock, whereas fiscal policy is made up of a countercyclical and debt-stabilizing public expenditure and of distortionary taxation on labor, dividends and interests on public bonds.

We find that: 1) in the presence of our monetary rule alone, domestic inflation variance falls more than in the only presence of fiscal rules, whereas output gap smoothing is stronger in the only presence of fiscal rules; 2) the combination of our monetary rule and fiscal rules reduces welfare losses more than the same rules singly considered.

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1 Introduction

The Stability and Growth Pact (henceforth SGP) with its budget rules represents a sort of ex-post fiscal coordination mechanism for EMU countries, where there is the "cohabitation" of one independent monetary policy and many fiscal policies. The aim of these fiscal rules is to stabilize Public Debt with respect to GDP, through the control of deficit with respect to GDP, both in the short term and in the medium term. In words, the SGP is founded on the idea that excessive deficits and high debts with respect to GDP are able to destroy the economic architecture of the EMU.

A large part of the literature (Buti et al. (1997), Melitz (2000), Wyplosz (2002), Galì and Perotti (2003)) has developed different works to understand whether the SGP has tied the EMU members' hands in pursuing the stabilization of the business cycles through the instruments of fiscal policy (i.e. taxation and public expenditure). Nevertheless, evidence is not univocal and covers a very short time span (at the most ten years) for the construction of a representative sample in the EMU that incorporates different scenarios for the business cycle and the application of fiscal policy instruments to it.

Buti et al. (1997) underline an excessive procyclicality for fiscal policy in EMU countries during and after severe recessions in the period 1961-1996; Melitz (2000) finds strong evidence of stabilization for the ratio Debt/GDP in EMU countries, but finds a weaker stabilizing movement for public expenditures; Wyplosz (2002) finds evidence for Italy, France and Germany of a countercyclicality in public consumption and an acyclicity (Italy) or procyclicality (France and Germany) for tax revenues. Anyway for the years 1992-2001¹ he finds some evidence of an asymmetric behavior of fiscal policy components: in France there is a countercyclical reaction to downswings and a procyclical reaction to upswings; in Italy, public expenditure is more countercyclical and taxation more procyclical during downswings. The study of the sub-sample 1992-2001 has been driven by a key question of a large part of literature: have the Maastricht convergence criteria and the SGP requirements weakened the stabilizing role of fiscal policy in EMU countries? An attempt to answer this question, supported by a detailed empirical analysis, is given by Galì and Perotti (2003), who point out that fiscal policy in the EMU has become more countercyclical over time, following what appears to be a trend that affects other industrialized EMU and non-EMU countries as well; therefore, the SGP constraints would not represent an impediment on a stabilization path. Moreover, the decline in public investment, observed in the recent data, seems to follow a common tendency in other countries and starts before the implementation of the SGP. However, as Galì and Perotti state, real recessions in the period after-Maastricht have been quite rare and hence the available data are not so binding to announce a "failure" in the stabilizing activity of fiscal policy.

The goal of this paper is to evaluate, within a DSGE model, the performances of two fiscal rules, the former on public expenditure and the latter on taxation,

¹The global sample covers the period 1971-2001.

and one monetary rule, i.e. a Taylor rule increased by a stochastic component driven by a monetary shock. Public expenditure follows a countercyclical and debt-stabilizing rule, whereas taxation is given by the sum of tax revenues coming from distortionary taxation on labor, on dividends and interests on public bonds issued by the government to finance its stock of debt. In this way, public expenditure pursues both the objectives of debt stabilization, as implicitly required by the SGP from fiscal policy, and business cycle stabilization, that is one of the purposes assigned to fiscal policy (Musgrave (1959))².

Our analysis makes use of a microfounded theoretical New-Keynesian model with sticky prices à la Calvo (1983) applied to a currency union, following a large body of literature (Beetsma and Jensen (2005), Ferrero (2005), Galì and Monacelli (2008), Colciago et al. (2008) among others), and a welfare loss function for each country belonging to the currency union to compute the consumers' losses in the presence of monetary and fiscal rules. This approach has been implemented by much literature for monetary policy rules (Rotemberg and Woodford (1999) and Galì (2008) among others), relying on a second-order approximation to the utility losses of the households caused by deviations of variables from their efficient allocation values. Also Ferrero (2005) uses a welfare-based approach to evaluate the desirability of fiscal and monetary rules in a currency union, his analysis differs from ours essentially for the presence of a rule on real stock of public debt instead of a rule on public consumption as we do and for the use of a different welfare loss function. Indeed, Ferrero (2005) uses Benigno and Woodford's (2005) welfare loss function, that is able to take into account the presence of distortionary taxation and a positive stock of debt with corresponding steady state values different from the ones of the Central Planner's solution. The welfare loss function adopted in this paper, instead, has the same structure as the one of Galì and Monacelli (2008)³: the arguments of this function are the squared domestic inflation, the squared output gap and the squared fiscal gap⁴. The benchmark value of these variables against which we measure the losses is represented by a fully flexible prices equilibrium with lump-sum taxation able to finance public consumption, whose optimal behavior is described in the Appendix, in the absence of public debt and any rules on monetary policy. In such a framework the fully flexible prices equilibrium calculated as the solution of

²This theory designs three purposes for fiscal policy and it also does the same for public expenditure: the provision for social goods, i.e. the *allocation function* of budget policy; the distribution of wealth among the citizens to equalize the incomes, i.e. the *distribution function* and the business cycle stabilization, i.e. the *stabilization function*.

³Galì and Monacelli (2008) show that, in the presence of sticky prices, the combined monetary-fiscal policy mix able to maximize the average welfare of union households must lead *at the union level* to a constant (zero) value both for the output gap, inflation and fiscal gap. Anyway, the same authors argue that the union-wide equilibrium in general cannot be an equilibrium under the optimal policy *for each member country*: in this case the second-best allocation of the inflation gap and the output gap will have a non trivial equilibrium dynamics as for the union level. This equilibrium dynamics can be described through a series of dynamics simulations, given an appropriate calibration for the model parameters.

⁴Fiscal gap is defined as the share of output used for public consumption less the amount of public expenditure to which the households give a weight in the utility function.

the Social Planner's problem is also supported at a decentralized level⁵.

In particular, we compare three different scenarios: in the first one there is the only presence of our monetary rule with lump-sum taxation able to finance public consumption at its optimal level, and in the absence of public debt; in the second scenario there are only fiscal rules and no monetary policy rules, whereas in the third scenario both fiscal and monetary rules are present.

Here is an overview of the results:

- the presence of only our monetary rule is able to generate a stronger decrease in domestic inflation variance than the presence of fiscal rules only;
- the presence of our fiscal rules generates an output gap smoothing stronger than our monetary rule alone;
- the combination of monetary and fiscal rules generates less welfare losses than the rules considered in isolation.

Hence, in the EMU the attainment of price stability should depend on the common monetary policy, whereas fiscal policy, institutionally decentralized at a country level, should be focused on output gap stabilization, that, in turn, can be also reached in the presence of rules that ensure countercyclicality to fiscal policy, together with debt stabilization, as required by the SGP.

The paper is organized as follows. The model structure and its properties are set out in section 2. In Section 3, we derive the equilibrium market clearing conditions for the demand side of the market and for the supply side; in section 4, we discuss the calibration of the model parameters, whereas in section 5 we analyse, through the impulse response functions, the results coming from the time series generated by the model both in the presence of a technology shock and a union-wide interest rate shock. In section 6, we conclude.

2 A Currency Union Model for Fiscal Policy

We develop a closed currency union model, in the spirit of Galì and Monacelli (2008), made up of a continuum of small open economies, represented by the unit interval, such as the domestic policy decisions do not have any effect on the rest of the union.

We consider such a framework suitable to give a realistic description of the inner structure of a monetary union like the EMU, made up of fifteen members (each one with an independent fiscal authority). In fact and in line with a small country model, EMU countries are small relative to the union as a whole. Hence, each country's policy decisions have very little impact on the other countries; this context, as a matter of principle, could be described by widening the existing one to incorporate fifteen countries, but such undertaking would render the resulting model intractable.

⁵The analytical conditions are reported in the Appendix.

In this model, each country has identical preferences, technology and market structure; three agents are considered within each economy: the households, the firms and the government.

2.1 Households

All households living in the representative country i belonging to the monetary union aim to maximize an utility function defined over private consumption, C_t^i , hours of work, N_t^i and public expenditure G_t^i :

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^i, N_t^i, G_t^i) \quad (1)$$

The consumption index C_t^i is defined as the Dixit-Stiglitz aggregator over the bundles of goods produced in country i and f respectively:

$$C_t^i = \frac{(C_{i,t}^i)^{1-m} (C_{F,t}^i)^m}{(1-m)^{(1-m)} m^m} \quad (2)$$

where $C_{i,t}^i$ is a Dixit-Stiglitz aggregator defined over the continuum of differentiated goods produced in country i and $j \in [0, 1]$ denotes the type of good (within the set produced in country i). Each lot of goods is produced by a separate firm and no goods are produced in more than one country:

$$C_{i,t}^i = \left(\int_0^1 C_{i,t}^i(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \quad (3)$$

The aggregator $C_{F,t}^i$, in turn, is an index of country i 's consumption of imported goods and represents an exogenous variable for country i :

$$C_{F,t}^i = \exp \int_0^1 c_{f,t}^i df$$

where $c_{f,t}^i$ ⁶ = $\log C_{f,t}^i$ is the log of an index of the goods consumed by country i 's households that are produced in country f . This index is defined symmetrically to (3):

$$C_{f,t}^i = \left(\int_0^1 C_{f,t}^i(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}$$

Note that in the specification of composite consumption index above, $m \in [0, 1]$ is the weight of imported goods in the utility of private consumption; we can think at m as an index of openness. The parameter $\sigma > 1$ is the elasticity of substitution across goods produced within one country.

⁶In what follows, all the variables in small letters indicate the logarithms of the correspondent variables in capital letters.

The utility function takes the following form, following Galì and Monacelli (2008):

$$U^i (C_t^i, N_t^i, G_t^i) = (1 - \alpha) \log C_t^i + \alpha \log G_t^i - \frac{N_t^{i(1+\gamma)}}{(1 + \gamma)} \quad (4)$$

where parameter $\alpha \in [0; 1]$ represents the preference for public consumption G_t^i .

All the prices are set in the common numeraire. The law of one price is assumed to hold, so that the price of each variety of goods is the same across countries. The implied overall consumption-based price index is:

$$P_{c,t}^i = (P_t^i)^{1-m} (P_t^*)^m \quad (5)$$

where $P_t^i = \left(\int_0^1 P_t^i(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}$ represents country i 's domestic price index (i.e., an index of prices of domestically produced goods), for all $i \in [0, 1]$ and $P_t^* = \exp \int_0^1 p_t^f df$ is the union-wide price index, that from the viewpoint of any individual country can be seen as the price index for imported goods. Symmetrically, the price index for the basket of goods imported from country f is defined as $P_t^f = \left(\int_0^1 P_t^f(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}$. Note that the elasticities of domestic price index and union-wide price index defined in (5) correspond to the relative weights of the respective goods in the consumption basket.

The maximization of (1) is subject to the following sequence of budget constraints:

$$\begin{aligned} & \int_0^1 P_t^i(j) C_{i,t}^i(j) dj + \int_0^1 \int_0^1 P_t^f(j) C_{f,t}^i(j) dj df + E_t \left[Q_{t,t+1} \left(B_{i,t+1}^i + \int_0^1 B_{f,t+1}^i df + \Gamma_{i,t+1}^i + \int_0^1 \Gamma_{f,t+1}^i df \right) \right] \\ & \leq (1 - \tau_n) W_t^i N_t^i + B_{i,t}^i + \int_0^1 B_{f,t}^i df + \Gamma_{i,t}^i + \int_0^1 \Gamma_{f,t}^i df - \tau_t^i \end{aligned} \quad (6)$$

where $P_t^i(j)$ is the price of the domestic goods (expressed in units of the single currency) and $P_t^f(j)$ is the price of the imported goods. $B_{i,t}^i$ is the nominal value net from interest rate taxation in period t of a bond issued by the government of country i in period t and purchased by the citizens of country i , $B_{f,t}^i$ is the nominal value net from interest rate taxation in period t of a bond issued by the government of country f in period t and purchased by the citizens of country i :

$$\begin{aligned} B_{i,t}^i &= (1 - \vartheta) \left[1 + r_t^* (1 - \tau_k) D_t^i \right] \\ B_{f,t}^i &= \vartheta \left[1 + r_t^* (1 - \tau_k) D_t^f \right] \end{aligned}$$

where $D_t^i \left(D_t^f \right)$ is the nominal stock of public debt issued by the Government of country i (f) and whose law of motion is discussed later, r_t^* is the nominal interest rate for the currency union, that follows a Taylor rule as explained in the next section, the fraction $\vartheta(1 - \vartheta)$ is the share of foreign (domestic) debt purchased by the households of country i . $\Gamma_{i,t}^i$ is the nominal dividend in period

t for a share⁷ of a domestic firm purchased in t by the domestic households, whereas $\Gamma_{f,t}^i$ is the nominal dividend in period t for a share of a foreign firm purchased in t by the domestic inhabitants:

$$\Gamma_{i,t+1}^i = (1 - \nu)(1 - \tau_k) \Upsilon_t^i \quad (7)$$

$$\Gamma_{f,t+1}^i = \nu(1 - \tau_k) \Upsilon_t^f \quad (8)$$

where Υ_t^i (Υ_t^f) is the nominal profit of the representative firm of country i (f), the fraction $\nu(1 - \nu)$ is the share of domestic (foreign) firm held by each inhabitant of country i . τ_t^i denotes lump-sum taxes, whose role will be discussed later. W_t^i is the nominal wage, τ_n is a distortionary wage tax levied by the government on labor and τ_k is a distortionary tax levied by the government on the income coming both from dividends and bonds, that are the financial assets of the representative consumer. $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal values of each financial asset: it is common across countries. For each country, household's consumption must be optimally allocated across all differentiated goods: expenditure on goods j is negatively related to the relative price of goods j and it satisfies the following first-order conditions:

$$C_{i,t}^i(j) = \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\sigma} C_{i,t}^i; C_{f,t}^i(j) = \left(\frac{P_t^f(j)}{P_t^f} \right)^{-\sigma} C_{f,t}^i \quad (9)$$

for all $i, f, j \in [0, 1]$. it follows from the previous relationship that $\int_0^1 P_t^i(j) C_{i,t}^i(j) dj = P_t^i C_{i,t}^i$ and $\int_0^1 P_t^f(j) C_{f,t}^i(j) dj = P_t^f C_{f,t}^i$.

For country f symmetric conditions hold as for country i ; in particular, the consumer price index (CPI) is thus obtained:

$$P_{c,t}^f = \left(P_t^f \right)^{1-m} \left(P_t^* \right)^m \quad (10)$$

if we log-linearize and integrate both the members of the previous relationship over $f \in [0, 1]$, we obtain the equality (11):

$$p_{c,t}^* = p_t^* \quad (11)$$

Moreover, the optimal allocation of expenditures for imported goods by country of origin implies:

$$P_t^f C_{f,t}^i = P_t^* C_{F,t}^i \quad (12)$$

for all $f \in [0, 1]$.

Finally, combining all the previous results, we can express respectively the optimal allocation of expenditures between domestic and imported goods in

⁷In what follows, we suppose that the value of the shares held is given only by the value of the dividends; the initial value of each share is zero and there is not any form of capital gain.

country i and the total consumption expenditures by country i 's households in the following way:

$$P_t^i C_{i,t}^i = (1 - m) P_{c,t}^i C_t^i \quad (13)$$

$$P_t^* C_{F,t}^i = m P_{c,t}^i C_t^i \quad (14)$$

$$P_t^i C_{i,t}^i + P_t^* C_{F,t}^i = P_{c,t}^i C_t^i \quad (15)$$

Hence, the period budget constraint can be rewritten in a more compact way as:

$$P_{c,t}^i C_t^i + (1 - \tau_k) E_t \{Q_{t,t+1} F_{t+1}^i\} \leq (1 - \tau_k) F_t^i + (1 - \tau_n) W_t^i N_t^i$$

where F_t^i is a portfolio that collects all the financial assets purchased by the domestic households.

The remaining optimality condition for the control variables (C_t^i, N_t^i) for the households are given by:

$$C_t^i * (N_t^i)^\gamma = (1 - \alpha) \frac{W_t^i (1 - \tau_n)}{P_{c,t}^i} \quad (16)$$

For each household, the optimality condition for the allocation of wealth among the financial assets characterizes the stochastic discount factor as:

$$\beta \left\{ \left(\frac{C_t^i}{C_{t+1}^i} \right) \left(\frac{P_{c,t}^i}{P_{c,t+1}^i} \right) \right\} = Q_{t,t+1} \quad (17)$$

if we take the expected value on both the members of (20), we obtain the standard form of the Euler equation:

$$E_t \left[\beta \left(\frac{C_t^i}{C_{t+1}^i} \right) \left(\frac{P_{c,t}^i}{P_{c,t+1}^i} \right) R_{t+1}^* \right] = 1 \quad (18)$$

where $R_{t+1}^* = \frac{1}{E_t \{Q_{t,t+1}\}}$ is the gross nominal interest rate.

The assumption of complete markets for the financial assets across the union implies for country f an Euler equation analogous to (21):

$$E_t \left[\beta \left(\frac{C_t^f}{C_{t+1}^f} \right) \left(\frac{P_{c,t}^f}{P_{c,t+1}^f} \right) R_{t+1}^* \right] = 1 \quad (19)$$

Combining (18) and (19), we obtain:

$$C_t^i = \xi_i C_t^f \left(\frac{P_{c,t}^f}{P_{c,t}^i} \right) \quad (20)$$

where $\xi_i = \frac{P_{c,t+1}^i C_{t+1}^i}{P_{c,t+1}^f C_{t+1}^f}$ is a constant which depends on initial conditions regarding relative net asset positions: if we suppose zero net foreign asset holdings

for all countries together with the hypothesis of an ex-ante identical environment, we have the case in which $\xi_i = \xi = 1$ for all $i \in [0, 1]$, i.e.:

$$C_t^i = C_t^f \left(\frac{P_{c,t}^f}{P_{c,t}^i} \right) \quad (21)$$

Moreover, in each country government's assets are also subject to the following transversality condition:

$$\lim_{T \rightarrow \infty} E_t \left\{ Q_{t,T+1} \left[B_{i,T+1}^i + B_{i,T+1}^f \right] \right\} = 0 \quad (22)$$

$$\lim_{T \rightarrow \infty} E_t \left\{ Q_{t,T+1} \left[B_{f,T+1}^i + B_{f,T+1}^f \right] \right\} = 0 \quad (23)$$

In the next subsection, we describe the behavior of the central bank, that determines the monetary policy for the currency area, using the short-term nominal interest rate as its main instrument, but, before this step, we define the domestic CPI gross rate of inflation and the union-wide gross rate of inflation, respectively as:

$$\Pi_{c,t}^i = \frac{P_{c,t}^i}{P_{c,t-1}^i} \quad (24)$$

$$\Pi_t^* = \frac{P_t^*}{P_{t-1}^*} \quad (25)$$

2.2 Interest rate and monetary policy

The Central bank sets the short-term nominal interest rate r_t^* for the currency union as a linear function of the the union-wide current inflation π_t^* and the union-wide output gap, defined here as the deviation of output y_t^* from its level under fully flexible price value y_t^{*n} (Taylor rule). it is typically assumed that the coefficient on inflation ϕ_π is greater than one⁸, which implies that the central bank raises the nominal interest rate more than one-for-one in response to an increase in inflation:

$$r_t^* = \bar{r}^* + \phi_\pi (\pi_t^*) + \phi_y (y_t^* - y_t^{*n}) + \varpi_t \quad (26)$$

The quantity ϖ_t is an exogenous stochastic component, which follows the following homoschedastic white-noise process:

$$\varpi_t = \rho_\varpi \varpi_{t-1} + \varepsilon_{\varpi t}^*$$

we can think of $\varepsilon_{\varpi t}^*$ as a monetary shock, whereas \bar{r}^* is the steady-state level of long-term real interest rate, that can be derived from the Euler equation (21):

$$[\beta (1 + \bar{r}^*)] = 1 \quad (27)$$

$$\bar{r}^* = 1 - \frac{1}{\beta} \quad (28)$$

⁸This condition is in line with the findings of Bullard and Mitra (2001) and satisfies the so called "Taylor principle".

2.3 Firms

Each country has a continuum of firms represented by the interval $j \in [0, 1]$. Each firm produces differentiated goods with a linear technology:

$$Y_t^i(j) = A_t^i N_t^i(j) \quad (29)$$

where A_t^i is a country-specific aggregate technology index, whose law of motion follows an AR(1) process (in logs):

$$a_t^i = \rho_a a_{t-1}^i + \varepsilon_{at}^i \quad (30)$$

and $\rho_a \in \{0, 1\}$. Moreover, we suppose that the stochastic process, that generates labor productivity is an homoscedastic white noise and it's assumed a null correlation between the monetary shock and the technology shock.

The assumption of a linear technology implies that real marginal costs are given by:

$$mc_t^i = \log(1 - s^i) + w_t^i - p_t^i - a_t^i$$

where s^i is a constant employment subsidy with the role to offset firms' market power represented by the monopolistic competition. This subsidy is completely financed by the lump-sum taxes τ_t^i as in the model of Galí and Monacelli (2008).

We assume a staggered price setting à la Calvo (1983). As in Galí (2008), there is a number $1 - \theta$ of (randomly selected) firms, which sets new prices each period, with an individual firm's probability of reoptimizing in any given period being independent of the time elapsed since its last price resetting. Hence, the parameter θ is an index of stickiness. The aggregate price dynamics is described by the following equation:

$$\Pi_t^{i(1-\sigma)} = \theta + (1 - \theta) \left(\frac{P_t^{iR}}{P_{t-1}^i} \right)^{1-\sigma} \quad (31)$$

where $\Pi_t^i = \frac{P_t^i}{P_{t-1}^i}$ is the gross inflation rate and P_t^{iR} is the price set in period t by firms reoptimizing their price in that period. A firm reoptimizing in period t chooses a price P_t^{iR} that maximizes the current market value of the profits Υ_t^i , by solving the following problem:

$$\max_{P_t^{iR}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k}^i \left(P_t^{iR} Y_{t+k|t}^i - \Psi_{t+k}^i \left(Y_{t+k|t}^i \right) \right) \right\} \quad (32)$$

subject to the sequence of demand constraints

$$Y_{t+k|t}^i = \left(\frac{P_{t+k}^{iR}}{P_{t+k}^i} \right)^{-\sigma} C_{t+k}^i \quad (33)$$

for $k = 0, 1, 2, \dots$ and where $Q_{t,t+k}^i = \beta^k (C_{t+k}^i / C_t^i) (P_t^i / P_{t+k}^i)$ is the discount factor, $\Psi_t^i(\cdot)$ is the cost function of the firm, whereas $Y_{t+k|t}^i$ represents output

in period $t + k$ for a firm resetting its price in period t . Next, the first order condition associated with the problem (32) is given by:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k}^i Y_{t+k|t} \left(P_t^{iR} - M \psi_{t+k|t}^i \right) \right\} = 0 \quad (34)$$

where $\psi_{t+k|t}^i = \Psi_{t+k}^i \left(Y_{t+k|t}^i \right)$ indicates the nominal marginal cost in period $t + k$ for a firm resetting its price in period t and $M = \frac{\sigma}{\sigma-1}$ ⁹ is the optimal markup in absence of constraints on the frequency of price adjustment. Note that in the absence of price rigidities ($\theta = 0$) the previous condition collapses to the optimal price setting condition under flexible prices:

$$P_t^{iR} = M \psi_{t|t}^i \quad (35)$$

Moreover, in this particular case, by setting $s^i = \frac{1}{\sigma}$ and substituting this value and the definition of nominal marginal costs into (35), an optimal market allocation, that is able to completely eliminate the consequences of monopolistic competition, can be reached. In fact, if $s^i = \frac{1}{\sigma}$, expression (35) turns into the optimality condition of perfect competition, according to which the price should be equal to the marginal cost.

Then, we divide both the members of (34) by P_{t-1}^i :

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k}^i Y_{t+k|t} \left(\frac{P_t^{iR}}{P_{t-1}^i} - M * MC_{t+k|t} \Pi_{t-1,t+k}^i \right) \right\} = 0 \quad (36)$$

where $MC_{t+k|t} = \frac{\psi_{t+k|t}^i}{P_{t+k}^i}$ is the real marginal cost in period $t + k$ for firms whose last price set is in period t and, finally, we log-linearize the optimal price setting condition (36) around the zero inflation steady state with a first-order Taylor expansion:

$$p_t^{iR} - p_{t-1}^i = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[\widehat{mc}_{t+k|t}^i + (p_{t+k}^i - p_{t-1}^i) \right] \quad (37)$$

where $\widehat{mc}_{t+k|t}^i = mc_{t+k|t}^i - \overline{mc}$ is the logdeviation of marginal cost from its steady state value.

The optimal price setting strategy for the typical firm resetting its price in period t can be derived from (37), reducing some algebra:

$$p_t^{iR} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[mc_{t+k|t}^i + p_{t+k}^i \right] \quad (38)$$

with $\mu = \log \frac{\sigma}{\sigma-1}$, that represents the optimal markup in the absence of constraints on the frequency of price adjustment ($\theta = 0$). Hence, the price setting rule for the firms resetting their prices is represented by a charge over the

⁹The analytical derivation is reported in the Appendix.

optimal markup in the presence of fully flexible prices, given by a weighted average of their current and expected nominal marginal costs, with the weights being proportional to the probability of the price remaining effective $(\theta)^k$. In a zero-inflation steady state equilibrium and in the absence of price stickiness for all the firms ($\theta = 0$), the previous expression collapses to:

$$\bar{p}^i = \mu + 1 \quad (39)$$

Note that, under the hypothesis of constant returns to scale, implicit in the production function of our model, the marginal cost is independent from the level of production, i.e. $mc_{t+k|t}^i = mc_{t+k}^i$ and, hence, common across firms; so, the expression (38) can be rewritten in the following way:

$$p_t^{iR} - p_{t-1}^i = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t [mc_{t+k}^i] + \sum_{k=0}^{\infty} (\beta\theta)^k E_t [\pi_{t+k}^i] \quad (40)$$

Moreover, the equation (40) can be expressed as the following difference expression:

$$p_t^{iR} - p_{t-1}^i = \beta\theta E_t [p_{t+1}^{iR} - p_t^i] + (1 - \beta\theta) \widehat{mc}_t^i + \pi_t^i \quad (41)$$

and combined with (31) in a log-linear form in order to obtain the domestic inflation equation:

$$\pi_t^i = \beta E_t [\pi_{t+1}^i] + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \widehat{mc}_t^i \quad (42)$$

The previous expression states that the current value of domestic inflation is positively related to the discounted expected value of the inflation of one period ahead and to the log-deviation of real marginal cost according to the degree of price stickiness captured by the parameter θ .

2.4 Government

Following the same structure of private consumption, country i 's public consumption index is given by

$$G_t^i = \left(\int_0^1 G_t^i(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \quad (43)$$

where $G_t^i(j)$ represents the quantity of domestic goods purchased by the government. In line with the empirical evidence (Trionfetti, 2000), we assume that government purchases are fully oriented towards domestic goods. Each government chooses optimally the composition of a Dixit-Stiglitz aggregator over all goods produced in its own country to minimize expenditure, yielding a structure of demand schedules analogous to those of private consumption:

$$G_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\sigma} G_t^i \quad (44)$$

Nevertheless, we want to focus our attention on the allocation of the aggregate level of primary public expenditure and we assume that out-of-steady-state government consumption is related to its lagged out-of-steady-state level, to the lagged out-of-steady-state real stock of public debt and to the present out-of-steady-state output according to the following rule

$$\hat{g}_t^i = \zeta \hat{g}_{t-1}^i - \kappa \hat{d}_{R,t-1}^i - \chi \hat{y}_t^i \quad (45)$$

where

$$\hat{d}_{R,t-1}^i = \hat{d}_{t-1}^i - \hat{p}_{t-1}^i$$

indicates the log deviation of real public debt, the parameter κ measures the magnitude of the feedback on debt effect and the coefficient χ indicates the countercyclicality of public consumption

The rule (45) is not a model-based rule, but it tries to capture both the phenomenon of debt stabilization, as implicitly required by the SGP criteria, and the business cycle stabilization, that is an important objective of fiscal policy *tout court*.

The presence of a feedback on debt component in a spending rule has been also adopted by Kirsanova and Wren-Lewis (2007); these authors examine the impact of different degrees of feedback on debt for public expenditure in an economy with nominal rigidities where monetary policy is optimal: using a welfare function, they find the optimal level of fiscal feedback, which represents a threshold above which optimal monetary policy becomes less active and fiscal feedback does stabilize inflationary shocks, but with a welfare reduction, whereas, below this cut-off value, monetary policy becomes strongly passive with a deterioration in welfare.

Schmitt-Grohe and Uribe (2006) use a rule similar to (45) for taxation, i.e. the out-of-steady-state level of taxation is an increasing function of the lagged out-of-steady-state level of public liabilities, together with a monetary rule whereby the change in the nominal interest rate is set as a function of its own lag, lagged output growth, and lagged deviations of inflation from target. The authors maximize a welfare function in the presence of these rules and compare this framework with the Ramsey optimal policy: they find that interest rate rules with a positive response to output can lead to significant welfare losses, whereas optimal fiscal policy is passive. The optimal monetary and fiscal rule combination is able to attain the same level of welfare as the Ramsey optimal policy.

Muscatelli et al. (2004), in an empirical evaluation of monetary-fiscal interactions, estimate a New Keynesian dynamic general equilibrium model with the presence of monetary and fiscal rules. The former is based on a forward-looking Taylor rule specification, whereas the latter is based on a spending rule and on a taxation rule; both of them are built so that the variables are allowed to respond to output, to the ratio between the lagged budget deficit on GDP and to a persistence component, as for (45).

Recently, Colciago et al. (2008) in a two-country New Keynesian DSGE model, incorporating non Ricardian consumers, to analyse the stabilizing role

of national fiscal policies in a currency union, build a spending rule very similar to (45) with a feedback-on-debt term to explicitly take into account the SGP criteria on debt.

Our rule on taxation is such that real tax revenues are given by collecting lump sum taxation, distortionary taxation on labor, on domestic and foreign dividends and on domestic and foreign interests on bonds:

$$T_{R,t}^i = \frac{\tau_t^i}{P_t^i} + \tau_n * \frac{W_t^i}{P_t^i} * N_t^i + \tau_k \left[r_t^* (1 - \vartheta) D_{R,t}^i + r_t^* \vartheta D_{R,t}^f + \frac{(1 - \nu) \Upsilon_t^i}{P_t^i} + \frac{\nu \Upsilon_t^f}{P_t^f} \right] \quad (46)$$

Hence, real taxation is increasing in the hours worked and real wages, in domestic and foreign real stock of public debt and in domestic and foreign amount of real profits.

The law of motion of public debt is described by the following equation:

$$D_{Ri,t-1}^i (1 + r_t^*) + P_t^i G_t^i - T_t^i = D_{i,t}^i \quad (47)$$

that states that the stock of public debt in each period is equal to the present value of the past stock of public debt increased by the primary deficit, given by the difference between public expenditure and taxation $(P_t^i G_t^i - T_t^i)$.

3 Equilibrium Dynamics

3.1 Aggregate Demand and Supply side

The market clearing conditions for the goods j in country i can be expressed in the following way:

$$Y_t^i(j) = C_{i,t}^i(j) + \int_0^1 C_{i,t}^f(j) df + G_t^i(j) \quad (48)$$

The previous relationship states that domestic production of good j can be allocated to domestic consumption, to foreign consumption (i.e. exports) and to public consumption. Then, using the definitions of $C_{i,t}^i(j)$, $C_{i,t}^f(j)$ and $G_t^i(j)$, we obtain:

$$\begin{aligned} Y_t^i(j) &= \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\sigma} \left[\frac{(1 - m) P_{c,t}^i C_t^i}{P_t^i} + m \int_0^1 C_t^f \left(\frac{P_{c,t}^f}{P_t^i} \right) df + G_t^i \right] \\ Y_t^i(j) &= \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\sigma} \left[\frac{1}{P_t^i} ((1 - m) P_{c,t}^i C_t^i + m P_t^* C_t^*) + G_t^i \right] \end{aligned} \quad (49)$$

where

$$C_t^* = \int_0^1 C_t^f df$$

If we plug the previous relationship into (21), we are able to express C_t^* as a function of the domestic consumption:

$$\begin{aligned} Y_t^i(j) &= \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\sigma} \left[\frac{1}{P_t^i} \left((1-m) P_{c,t}^i C_t^i + m C_t^i P_{c,t}^i \right) + G_t^i \right] \\ Y_t^i(j) &= \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\sigma} \left[\frac{P_{c,t}^i}{P_t^i} (C_t^i) + G_t^i \right] \end{aligned} \quad (50)$$

Finally, by plugging (50) into the definition of the aggregate output index for country i , we obtain an expression of the aggregate domestic output:

$$Y_t^i = \left[\frac{P_{c,t}^i}{P_t^i} (C_t^i) + G_t^i \right] \quad (51)$$

The term $\log(Y_t^i - G_t^i)$ can be expressed in a first-order Taylor expansion about the steady state by the next expression¹⁰:

$$\log(Y_t^i - G_t^i) = \log((1 - \varkappa_i) \bar{Y}^i) + \frac{1}{1 - \varkappa} (\hat{y}_t^i - \varkappa_i \hat{g}_t^i) \quad (52)$$

where $\varkappa_i = \frac{\bar{G}^i}{\bar{Y}^i}$ is the steady state government spending share.

From (52), rewriting (51) in a log-deviation from the steady state values and recalling the definition of the out-of-steady-state public expenditure, we are able to build the demand side of this economy:

$$\hat{y}_t^i = \left[(1 - \varkappa_i) (\hat{c}_t^i + m (p_{c,t}^i - p_t^i)) + \frac{1}{(1 + \chi)} (\zeta \hat{g}_{t-1}^i - \kappa \hat{d}_{R,t-1}^i) \right] \quad (53)$$

The previous equation establishes that domestic output is positively related to domestic consumption, to the terms of trade and to the lagged real public expenditure, whereas output is decreasing in the lagged real stock of public debt. The negative relationship between domestic output and the lagged real stock of public debt represents the amount of resources withdrawn from public consumption in order to reduce the lack of balance in public accounts. Furthermore, the higher is the steady state government spending share, the lower is the weight of private consumption in the determination of domestic output due to the crowding out effect of public expenditure.

To build the supply side of this economy, first we have to rewrite the dynamics of domestic inflation:

$$\pi_t^i = \beta E_t [\pi_{t+1}^i] + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \widehat{mc}_t^i \quad (54)$$

and recall the definition of marginal cost (in logs):

$$mc_t^i = \log(1 - s^i) + w_t^i - p_t^i - a_t^i \quad (55)$$

¹⁰The analytical derivation is reported in the Appendix.

Then we add and subtract to the right side of (55) the quantity $p_{c,t}^i$:

$$mc_t^i = \log(1 - s^i) + (w_t^i - p_{c,t}^i) + (p_{c,t}^i - p_t^i) - a_t^i$$

and combining the resulting expression with the previous results we obtain the next relationship:

$$mc_t^i = \log(1 - s^i) + c_t^i + \gamma n_t^i - \log(1 - \alpha) - \log(1 - \tau_n) + (p_{c,t}^i - p_t^i) - a_t^i \quad (56)$$

Note that, according to (51), the terms of trade $(p_{c,t}^i - p_t^i)$ are equal to $\log(Y_t^i - G_t^i) - c_t^i$, i.e.:

$$mc_t^i = \log(1 - s^i) + \gamma n_t^i - \log(1 - \alpha) - \log(1 - \tau_n) + \log(Y_t^i - G_t^i) - a_t^i \quad (57)$$

The combination of (57) and (52)¹¹ leads to a definition of real marginal cost expressed in logdeviation from the steady state value:

$$\widehat{mc}_t^i = \left(\frac{1}{1 - \varkappa_i} + \gamma \right) \hat{y}_t^i - \frac{\varkappa_i}{1 - \varkappa_i} \hat{g}_t^i - (1 + \gamma) \hat{a}_t^i \quad (58)$$

Given the level of output, an increase in government spending crowds out domestic consumption and generates a real appreciation: both these pressures lead to a reduction in real marginal cost, whose dimension is measured by the parameter \varkappa_i .

Finally, by plugging (58) into (54), we are able to derive the *new Keynesian Phillips curve* for the domestic economy:

$$\pi_t^i = \beta E_t [\pi_{t+1}^i] + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \left(\left(\frac{1}{1 - \varkappa_i} + \gamma \right) \hat{y}_t^i - \frac{\varkappa_i}{1 - \varkappa_i} \hat{g}_t^i - (1 + \gamma) \hat{a}_t^i \right) \quad (59)$$

and, by integrating (59) over $i \in [0, 1]$, we are able to obtain the corresponding *new Keynesian Phillips curve* for the union as a whole:

$$\pi_t^* = \beta E_t [\pi_{t+1}^*] + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \left(\left(\frac{1}{1 - \varkappa_i} + \gamma \right) \hat{y}_t^* - \frac{\varkappa_i}{1 - \varkappa_i} \hat{g}_t^* - (1 + \gamma) \hat{a}_t^* \right) \quad (60)$$

where

$$Y_t^* = C_t^* + G_t^* \quad (61)$$

$$a_t^* = \int_0^1 a_t^i dt \quad (62)$$

and all the other union-wide variables are defined in a symmetric way with respect to the country-specific one.

¹¹For this result we use the property that in a symmetric steady state the steady state value of the terms of trade is equal to one and so $\bar{p}_c^i - \bar{p}^i = 0$

3.2 The Efficient Allocation under Flexible Prices

Before describing the dynamic equilibrium conditions in the presence of nominal rigidities, we need to derive an expression for the flexible-price output Y_t^{in} , i.e. the natural level of output, in order to define a measure of the output-gap of each member's economy and then the one of the whole monetary union. Note that, due to the presence of distortionary taxation, the only way to calculate the natural level of output is the solution of the decentralized economy under flexible prices, because in this case, the equilibrium derived by the solution of the social planner's problem would not be supported in the decentralized context, as done by Galí and Monacelli (2008). In the absence of constraints on the frequency of price adjustment, the price setting rule follows the equation (35), i.e. each firm charges the price as a markup over the nominal marginal cost and the value of markup is optimal and equal to $M = \frac{\sigma}{\sigma-1}$ and $mc = \log\left(\frac{\sigma-1}{\sigma}\right)$. The procedure followed to characterize and derive the fully flexible price output Y_t^{in} is given by the solution of the decentralized economy, as before, with a null value for the price-stickiness parameter θ , i.e.:

$$mc^i = \log(1 - s^i) + \gamma n_t^i - \log(1 - \alpha) - \log(1 - \tau_n) + \log(Y_t^{in} - G_t^i) - a_t^i \quad (63)$$

If we express the previous expression in log-deviation from the steady state values, we obtain:

$$0 = \left(\frac{1}{1 - \varkappa_i} + \gamma\right) \hat{y}_t^{in} - \frac{\varkappa_i}{1 - \varkappa_i} \hat{g}_t^i - (1 + \gamma) \hat{a}_t^i \quad (64)$$

Finally, the closed solution for the fully flexible prices output for country i and for the currency union are given by:

$$\begin{aligned} \hat{y}_t^{in} &= \frac{\left(\frac{\varkappa_i}{1 - \varkappa_i} \hat{g}_t^i + (1 + \gamma) \hat{a}_t^i\right)}{\left(\frac{1}{1 - \varkappa_i} + \gamma\right)} \\ \hat{y}_t^{*n} &= \frac{\left(\frac{\varkappa_i}{1 - \varkappa_i} \hat{g}_t^* + (1 + \gamma) \hat{a}_t^*\right)}{\left(\frac{1}{1 - \varkappa_i} + \gamma\right)} \end{aligned}$$

Subtracting (64) from (58) obtains

$$\widehat{mc}_t^i = \left(\frac{1}{1 - \varkappa_i} + \gamma\right) \hat{y}_t^{in} \quad (65)$$

and similarly for the currency union:

$$\widehat{mc}_t^i = \left(\frac{1}{1 - \varkappa_i} + \gamma\right) \hat{y}_t^{*n} \quad (66)$$

where $\hat{y}_t^i = y_t^i - y_t^{in}$ and $\hat{y}_t^* = y_t^i - y_t^{*n}$ are respectively the definitions of domestic output gap and union-wide output gap.

The previous relationships state that the log-deviation of real marginal cost from the steady state is proportional to the log-deviation of output from its natural level, i.e. the *output gap*.

3.3 Calibration

Before describing the equilibrium behavior of the prototype member economy under the framework illustrated above, we need to give a numerical value to the parameters. To this purpose we distinguish two kinds of parameters: general parameters and fiscal policy parameters. The former $(m, \beta, \gamma, \sigma, \alpha, \rho, \phi, \bar{r}^*, \theta, \nu, \vartheta)$ are calibrated according to the benchmark parametrization adopted by Galì and Monacelli (2008) and some stylized facts about EMU countries, whereas for the latter we use the EMU data $(\kappa, \zeta, \tau_n, \tau_k)$. The following table summarizes the benchmark parametrization for general parameters:

Table 1 : Calibration for general parameters

<i>Parameter</i>	<i>Value</i>
m	0.4
β	0.99
γ	3.0
σ	6.0
α	0.25
ρ	0.95
ϕ_π	1.7
ϕ_y	0.125
\bar{r}^*	0.04
θ	0.75
ν	0.50
ϑ	0.56

The values calibrated for the labor supply elasticity (γ), for the elasticity of substitution between differentiated goods (σ), for the degree of stickiness of prices (θ), for the average share of public consumption (α) for EMU countries, for the subjective discount factor (β), which is in line with the real business cycle literature and implies the steady state value for interest rate (\bar{r}), ensure a stable solution to the model. Moreover, following the real business cycle literature (King and Rebelo (1999)), we suppose a high value for the persistence coefficient of total labor productivity (ρ). Finally, the index of openness with respect to EMU countries (m), the share of domestic debt held by domestic households ($1 - \vartheta$) and the fraction of domestic firms held by the residents ($1 - \nu$) are set to a value able to match statistical data about Euro-area balance of payments (source: IMF statistical data (sample 1995-2005)). Monetary policy parameters (\bar{r}, ϕ) are consistent with the empirical literature about Taylor rule in the EMU (Smets and Wouters (2003)) and are in line with the Taylor principle ($\phi_\pi > 1$).

In the next table we calibrate fiscal policy parameters:

Table 2 : Calibration for fiscal policy parameters

<i>Parameter</i>	<i>Value</i>
τ_n	0.35
τ_k	0.24
ζ	0.85
χ	0.51
κ	0.20

Tax rate on labor (τ_n) is set equal to the average annual implicit tax rate on labor employed (0.35) for the Euro area, tax rate on dividends (τ_k) is set equal to the average implicit tax rate on capital income (0.24) for the Euro area, whereas the persistence coefficient in public expenditure (ζ) is set equal to the average elasticity of real public expenditure with respect to its lagged value (0.85) for the Euro area (source: Eurostat statistical data (sample 1995-2005)). At the same time, we let the feedback on debt parameter (κ) assume a value equal to the average elasticity of real public expenditure with respect to lagged real stock of public debt for the Euro area (0.20) and the countercyclical parameter (χ) take a value equal to the elasticity of real public expenditure with respect to output (0.51) (source: Eurostat statistical data (sample 1995-2005)). In the next section we discuss the dynamic behavior of the model both in the presence of a technology shock and of a union-wide interest rate shock under the policy rules described above.

4 Dynamic simulations under the policy rules

In this section we discuss the dynamic equilibrium behavior of a representative member economy under the model discussed above, by resorting to a series of dynamic simulations with the parameterization described in the previous section. In particular, we focus our attention on the responses of fiscal policy instruments (public expenditure, tax revenues, public debt and the ratio public debt/GDP) to the domestic shock, i.e. the technology shock, and to the union-wide shock, i.e. interest rate shock.

The first set of figures (Section 7.4.1.) displays the dynamic response of output, output gap, real public debt, the ratio debt/GDP, total private consumption, real public expenditure, hours worked, real wages, real marginal costs, real tax revenues and real profits in the presence of a technology shock. The rigidity in aggregate demand resulting from the stickiness of the domestic price level leads technology shock (that in this framework is a labor productivity shock) to generate a negative comovement between hours worked and productivity. This result is also supported by strong empirical evidence (Galì (1999), Francis and Remy (2005); Christiano et al. (2003) among others). The intuition for this result is straightforward. When a technology shock hits the economy, the increase in productivity determines a fall in real marginal costs, domestic prices decrease, as a consequence of a right-shift in the aggregate supply curve and following the decrease in real marginal costs. Aggregate demand and private

consumption increase, due to the fall in domestic prices. Nevertheless, aggregate demand and prices, given the nominal rigidities, for which only a fraction $1 - \theta$ of firms reset their prices, change less than under fully flexible prices. Aggregate output increases, but less than in the absence of price rigidities: for this reason, a contraction in the output gap occurs. Furthermore, because labor is more productive with a consequent increase in the real wages, the firms will require less labor input. As a consequence, output does not increase in the same proportion of the productivity shock. On the other hand, real profits increase due both to the positive shift of output and to the contraction in real marginal costs. Real tax revenues are procyclical, i.e. with the calibration adopted they increase whenever output does the same. On the contrary, public debt is countercyclical, that is, every time a positive productivity shock hits the economy and shifts upwards domestic output, government debt shows a negative deviation from its steady state value: an increase in output, leading to an increase in real tax revenues, with a countercyclical public consumption, generates a countercyclical movement of the stock of public debt. In this way, the ratio debt/GDP decreases for some periods, due to the contemporaneous contraction of the real stock of public debt and the increase in output, and then reverts to the original steady state value. Thus, public expenditure, together with taxation, plays the role of a "smoother" for output cyclical fluctuations.

The second set of figures (Section 7.4.2.) displays the dynamic response of the same variables in the presence of a positive shock on the nominal interest rate. An increase in the union-wide interest rate determines a reduction in the rate of inflation and so in the level of prices (union-wide prices and domestic prices). The decrease in the price consumer index pushes up real wages and, at the same time, real marginal costs, that, with an invariant level of output and productivity, cause a contraction in labor demand and hence in hours worked. The reduction in labor input generates, as a consequence, a fall in output, in profits and in real tax revenues. Private consumption and output decrease less than under fully flexible prices due to the stickiness in prices; for this reason and because of the increase in real marginal cost, an increase in the output gap occurs. Real public debt confirms its countercyclicity as in the case of technology shock, i.e. in this case it goes up when output falls. Indeed, in this context the upper pressure of real public is strongly determined by the increase in the interest rate. In this case, the ratio debt/GDP increases for some periods, due to an expansion of the real stock of public debt and to a contraction in output, and then reverts to the original steady state value.

Hence, also in the presence of a union-wide monetary policy shock fiscal policy is able to stabilize output cyclical fluctuations through a procyclical taxation and a countercyclical public expenditure.

5 A Welfare Analysis

This section aims to evaluate fiscal and monetary rules' performances, basing on a welfare-criterion referred to country i and relying on a second-order ap-

proximation to the utility losses of the consumers. In order to measure these utility losses, we make use of a welfare function defined here as the discounted sum of the utilities across households:

$$F \equiv \sum_{t=0}^{\infty} \beta^t U(C_t^i, N_t^i, G_t^i) \quad (67)$$

The benchmark values against which we measure the welfare losses associated to our policy rules are referred to an economic framework without distortionary taxation, with zero stock of public debt, with lump-sum taxation able to finance public consumption at its optimal level and without monetary rules. In such an environment, output gap is measured as the difference between actual output and the fully flexible price one, with the latter also optimal from the social planner's point of view, due to the absence of distortionary taxation. For this purpose¹², we have to impose for each variable a steady state value corresponding to the one deriving from the solution of the Social Planner's problem¹³.

As already shown in Galí and Monacelli (2008) a second order approximation to (67) can be rewritten as the average utility losses of union households resulting from fluctuations about the efficient steady state in the following functional form¹⁴:

$$\Theta \simeq \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left(\frac{\sigma}{\alpha} (\pi_t^i)^2 + (1 + \gamma) (\tilde{y}_t^i)^2 + \frac{\alpha}{1 - \alpha} (\tilde{f}_t^i)^2 \right) + tips \quad (68)$$

where *tips* denotes terms that are independent of policy and $\tilde{f}_t^i = (g_t^i - y_t^i) - \log \alpha$ defined as fiscal gap. In words, this variable represents the share of output used for public consumption less the amount of public expenditure to which the households give a weight in the utility function ($\log \alpha$) and for this reason it represents an inefficient gap.

Taking the expected value on both side of (68) at time 0 obtains the average welfare loss per period given by the following linear combination of output gap and inflation variances and the variance of fiscal gap:

$$L = \frac{1}{2} \left[\frac{\sigma}{\alpha} var(\pi_t^i) + (1 + \gamma) var(\tilde{y}_t^i) + \frac{\alpha}{1 - \alpha} var(\tilde{f}_t^i) \right] \quad (69)$$

Using (69), given the monetary and fiscal rules together with a calibration for the model's parameters above described, it's possible to compute the second order moments of the simulated time series¹⁵ for output gap, inflation and fiscal gap, in order to derive the corresponding welfare losses associated to these rules.

¹²For this point we thank in particular Pierpaolo Benigno for his important suggestions.

¹³These values, calculated by Galí and Monacelli (2008), are reported in Appendix.

¹⁴For the analytical derivation of the welfare function, see Appendix in Galí and Monacelli (2008).

¹⁵The time series generated by the simulation process are HP-filtred.

Table 3 reports the measures of domestic inflation, output gap and fiscal gap variance together with the per cent contributions to welfare losses in round brackets: in column "A" we analyse the effects of the only presence of Taylor rule (26) with lump-sum taxation able to finance public consumption at its optimal level and zero public debt, in column "B" we show the effects of fiscal rules (45 and 46) with no rules for monetary policy, whereas column "C" evaluates the joint effects of the monetary and fiscal rules.

Table 3 : Contributions to Welfare Losses

	<i>Taylor Rule (A)</i>	<i>Fiscal Rules (B)</i>	<i>Taylor Rule + Fiscal Rules (C)</i>
$\frac{1}{2} \frac{\sigma}{\alpha} var(\pi_t^i)$	0.18 (28.57%)	0.87 (91.58%)	0.01 (25%)
$\frac{1}{2} (1 + \gamma) var(\hat{y}_t^i)$	0.20 (31.75%)	0.05 (5.26%)	0.02 (50%)
$\frac{1}{2} var(\hat{f}_t^i)$	0.25 (39.68%)	0.03 (3.16%)	0.01 (25%)
<i>Total</i>	0.63	0.95	0.04

From a first inspection of the table, it's evident how the mix of fiscal and monetary rules (Column C) reduces welfare losses more than the single rules and so it should be preferred. The comparison between fiscal rules and monetary rule shows that fiscal rules generate larger welfare losses than Taylor rule. Furthermore from an analysis of Column A and B it emerges that Taylor rule is able to better reduce the fluctuations in domestic inflation in comparison with fiscal rules, whereas both output gap and fiscal gap show smaller variations in the presence of fiscal rules than under Taylor rule. Finally, under the monetary rule, fiscal gap has the most important role in explaining welfare losses whereas under fiscal rules, this role belongs to domestic inflation.

From this picture two key results emerge: 1) the combination of a monetary policy, that positively responds to inflation and output gap, and fiscal rules made up of distortionary taxation on labor, dividends and interests on public bonds and of a countercyclical and debt-stabilizing public consumption is welfare improving than the same fiscal and monetary rules singly considered; 2) in the presence of our monetary rule domestic inflation variance falls more than in the presence of fiscal rules, whereas output and fiscal gap fluctuations are better smoothed by our fiscal rules on public expenditure and taxation. Therefore, in a currency union scenario like the EMU, the common monetary policy should mainly focus on inflation stabilization, whereas fiscal policy, institutionally decentralized at a country level, should centre on output gap stabilization. This last objective can be reached in the presence of fiscal rules focused not only on business cycle stabilization but also on debt stabilization, consistently with the SGP.

These findings have some similarities with those of Ferrero (2005). He finds that, in the presence of a monetary rule that positively reacts to inflation and output gap and with a fiscal constraint on real debt such that it positively reacts to output gap, such fiscal policy leads to welfare gains if compared to

balanced budget rules, whereas monetary policy better smoothes inflation and hence should focus on price stability.

6 Conclusions

This paper develops a New-Keynesian multicountry model applied to the EMU context with sticky prices and the presence of policy rules related to fiscal policy and monetary policy. The former is managed by the government sector, institutionally decentralized at a single country level, that makes use of distortionary taxation on labor, on dividends and interests on public bonds and of public consumption following a countercyclical and debt-stabilizing behavior. The latter is under the control of the common monetary authority, that follows a Taylor rule increased by a stochastic component driven by a monetary shock.

From a welfare analysis of the policy rules we have the chance to evaluate the welfare contribution, in terms of welfare losses, of the monetary rule in a scenario without public debt and with lump-sum taxation able to finance public consumption at its optimal level, of the fiscal rules in a context without any monetary rules and of the combination of fiscal rules and monetary rules.

The results obtained show that i) in the presence of our monetary rule alone, domestic inflation fluctuations are better smoothed than in the presence of our fiscal rules; ii) output gap variance is smaller in the presence of fiscal rules alone than whenever only the monetary rule is present; iii) the fiscal-monetary policy mix made up of our rules is able to lower welfare losses more than the monetary and fiscal rules in isolation.

The policy implications of these results are that i) in a currency union, like the EMU, monetary policy should have the objective of inflation stabilization, as institutionally indicated in the Maastricht Treaty; ii) fiscal policy should centre on output gap stabilization. This aim can be pursued in the presence of fiscal rules not only oriented to business cycle stabilization, that is one of the purposes assigned to fiscal policy by the theory of public finance (Musgrave (1959)), but also to debt stabilization, as prescribed by the SGP.

The theoretical structure described above calls for further analysis on several points. The model ignores capital accumulation and stickiness is only confined to prices and not to wages. Furthermore, it could be useful to make a distinction in the government expenditure rule between current public expenditure and capital public expenditure, in order to define different behaviors of these components.

We plan to further examine some of these points in a future work.

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7 Appendix

7.1 Profit maximization problem in steady state

$$\begin{aligned}
& \underset{\bar{Y}^i(j)}{Max} \left[\begin{aligned} & (\bar{Y}^i(j))^{\frac{\sigma-1}{\sigma}} * \bar{P}^i * \\ & * (\bar{Y}^i)^{-\frac{1}{\sigma}} - \frac{\bar{W}^i}{\bar{A}^i} \bar{Y}^i(j) \end{aligned} \right] \\
& \frac{\partial \bar{Y}^i(j)}{\partial \bar{Y}^i(j)} = 0 : \left(\frac{\sigma-1}{\sigma} \right) (\bar{Y}^i(j))^{-\frac{1}{\sigma}} * \bar{P}^i * \\
& * (\bar{Y}^i)^{-\frac{1}{\sigma}} - \frac{\bar{W}^i}{\bar{A}^i} = 0 \\
& \left(\frac{\sigma-1}{\sigma} \right) (\bar{Y}^i(j))^{-\frac{1}{\sigma}} * \bar{P}^i * (\bar{Y}^i)^{-\frac{1}{\sigma}} - \frac{\bar{W}^i}{\bar{A}^i} = 0 \\
& \left(\frac{\sigma-1}{\sigma} \right) (\bar{Y}^i(j))^{-\frac{1}{\sigma}} * \bar{P}^i * (\bar{Y}^i)^{-\frac{1}{\sigma}} = \frac{\bar{W}^i}{\bar{A}^i} \\
& \left(\frac{(\bar{Y}^i(j))}{(\bar{Y}^i)} \right)^{-\frac{1}{\sigma}} * \bar{P}^i * \left(\frac{\sigma-1}{\sigma} \right) = \frac{\bar{W}^i}{\bar{A}^i} \\
& \bar{P}^i(j) * \left(\frac{\sigma-1}{\sigma} \right) = \left(\frac{\bar{W}^i}{\bar{A}^i} \right) \\
& \bar{P}^i(j) = \left(\frac{\bar{W}^i}{\bar{A}^i} \right) \left(\frac{\sigma}{\sigma-1} \right) \\
& \left(\int_0^1 \bar{P}^i(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} = \left(\int_0^1 \left(\left(\frac{\bar{W}^i}{\bar{A}^i} \right) \frac{\sigma}{\sigma-1} \right)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \\
& \bar{P}^i = \left(\frac{\bar{W}^i}{\bar{A}^i} \right) \left(\frac{\sigma}{\sigma-1} \right) \tag{70} \\
& \frac{\bar{W}^i}{\bar{A}^i \bar{P}^i} = \left(\frac{\sigma-1}{\sigma} \right) = \overline{MC}^i \tag{71}
\end{aligned}$$

The expression (70) states that, in steady state, the level of price \bar{P}^i given by the product of marginal costs $\left(\frac{\bar{W}^i}{\bar{A}^i} \right)$ and markup $\left(\frac{\sigma}{\sigma-1} \right)$.

7.2 Taylor expansion of $\log(Y_t^i - G_t^i)$

Let $\varkappa_i = \frac{\bar{G}}{\bar{Y}}$ the steady state government spending share. Define $\hat{y}_t^i = \log \frac{Y_t^i}{\bar{Y}}$ and $\hat{g}_t^i = \log \frac{G_t^i}{\bar{G}}$. A first-order Taylor expansion of $\log(Y_t^i - G_t^i)$ about the steady state yields:

$$\begin{aligned}
\log(Y_t^i - G_t^i) &= \log((1 - \varkappa_i) \bar{Y}) + \frac{1}{1 - \varkappa_i} \left(\frac{Y_t^i - \bar{Y}}{\bar{Y}} \right) - \frac{\varkappa_i}{1 - \varkappa_i} \left(\frac{G_t^i - \bar{G}}{\bar{G}} \right) \\
\log(Y_t^i - G_t^i) &= \log((1 - \varkappa_i) \bar{Y}) + \frac{1}{1 - \varkappa_i} (\hat{y}_t^i - \varkappa_i \hat{g}_t^i)
\end{aligned}$$

7.3 The efficient steady-state derived by the solution of Central Planner's problem

The symmetric steady state implied by the solution of the Social Planner's problem is the same as the one of Galí and Monacelli (2008). In this context taxation takes the only form of lump-sum taxes and public debt is absent:

$$\bar{N}^i = 1 \quad (72)$$

$$\bar{Y}^i = \bar{A}^i \quad (73)$$

$$\bar{C}_i^i = (1 - \alpha)(1 - m)\bar{A}^i \quad (74)$$

$$\bar{C}_i^f = (1 - \alpha)m\bar{A}^i \quad (75)$$

$$\bar{G}^i = \alpha\bar{A}^i \quad (76)$$

$$\bar{C}^i = (1 - \alpha)(\bar{A}^i)^{1-m}(\bar{A}^*)^m \quad (77)$$

$$\bar{Y}^* = \bar{A}^* \quad (78)$$

$$\bar{C}^* = (1 - \alpha)(\bar{A}^*) \quad (79)$$

$$\bar{G}^* = \alpha\bar{A}^* \quad (80)$$

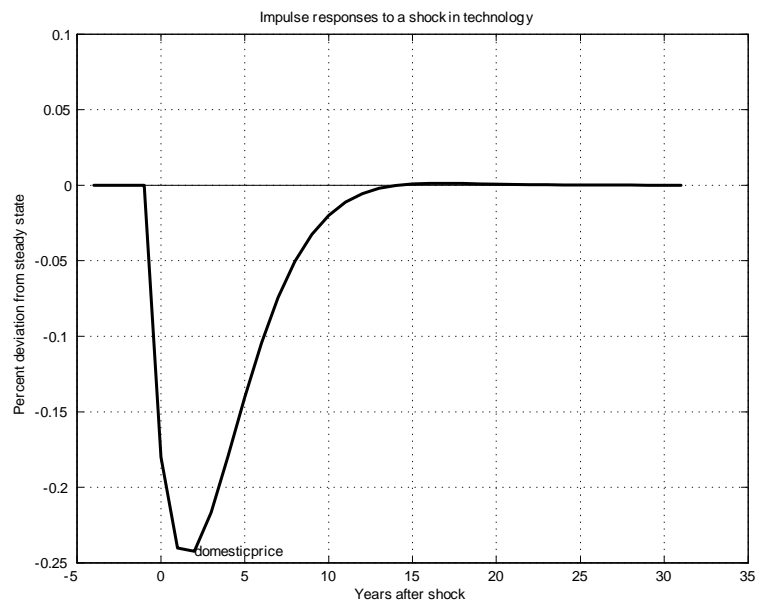
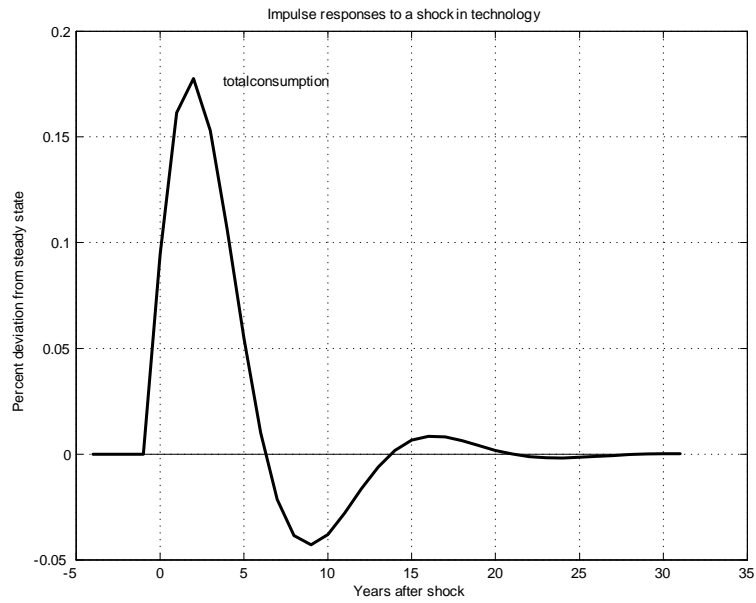
The previous conditions are supported as a fully flexible prices equilibrium at a decentralized level with the subsidy being equal to $\frac{1}{\sigma}$, that is the value able to completely offset the market distortions deriving by the monopolistic competition. Moreover, the efficient allocations of the terms of trade is given by:

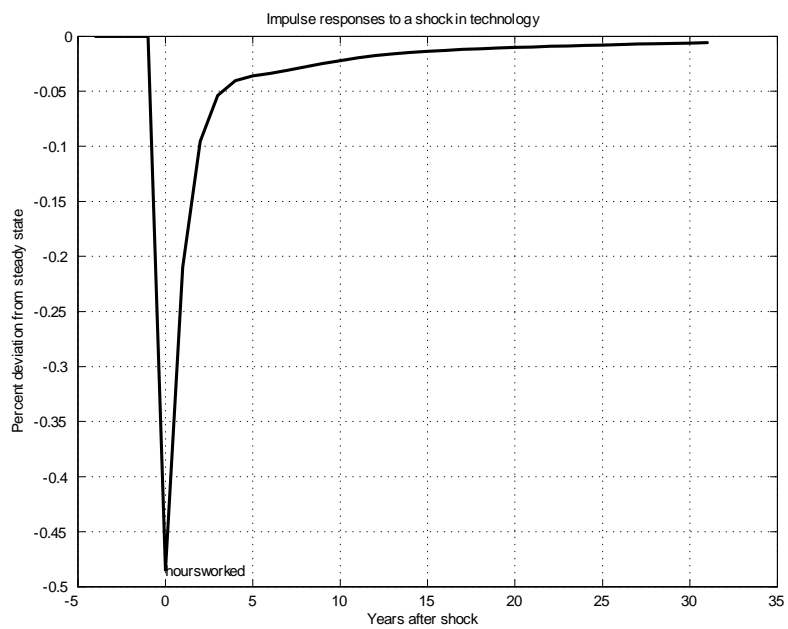
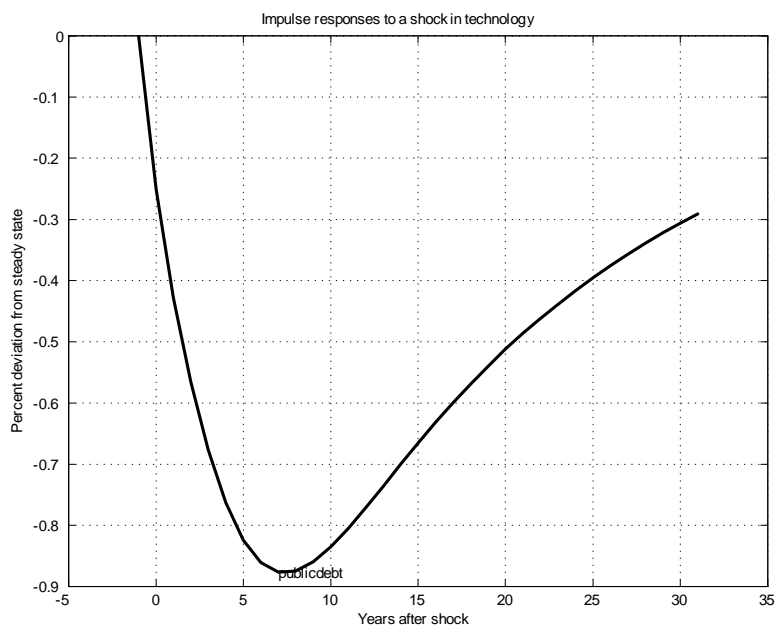
$$\left(\frac{\bar{P}_c^i}{\bar{P}^i}\right)^m = \left(\frac{\bar{C}^i}{\bar{C}^*}\right)^{\frac{1}{1-m}} = \frac{\bar{A}^i}{\bar{A}^*}$$

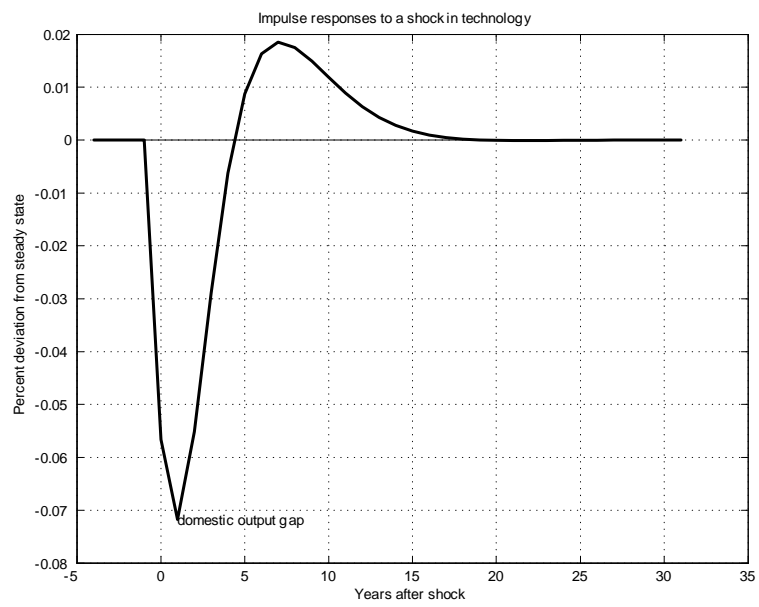
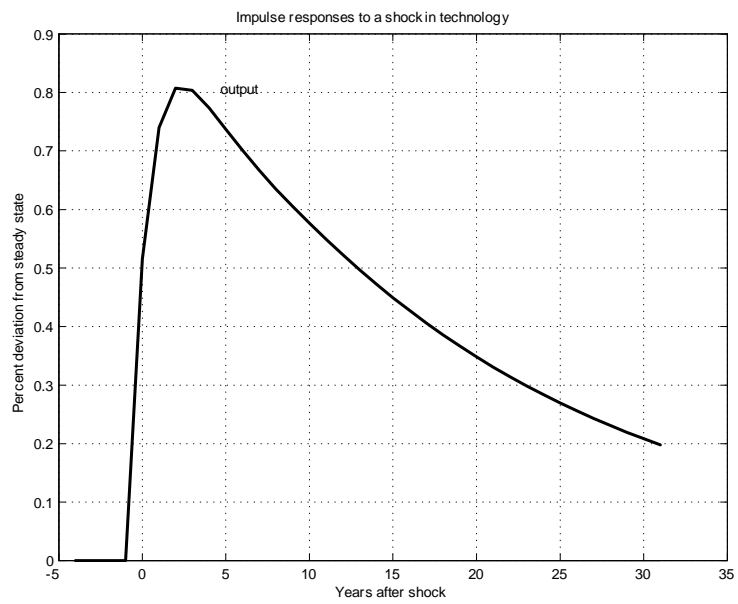
All these conditions with the time subscript represent the Social Planner's dynamic equilibrium used as a benchmark to evaluate the policy rules.

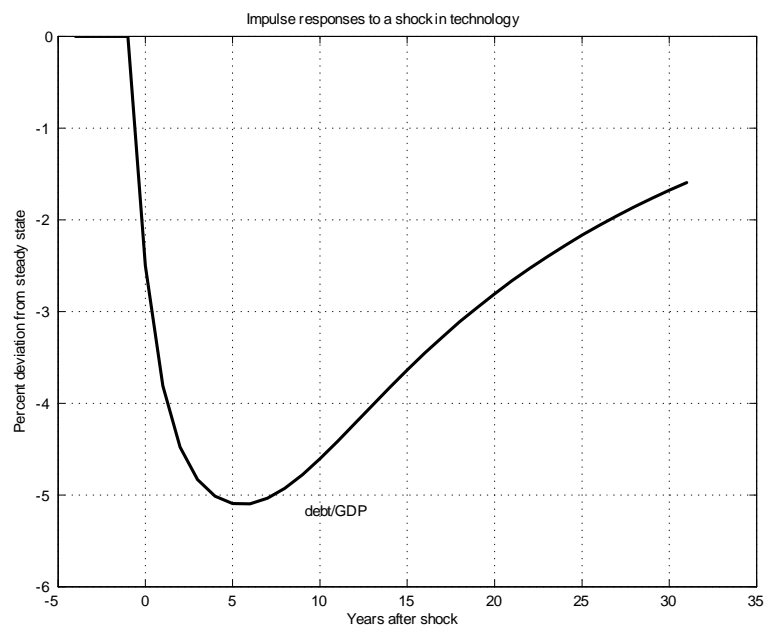
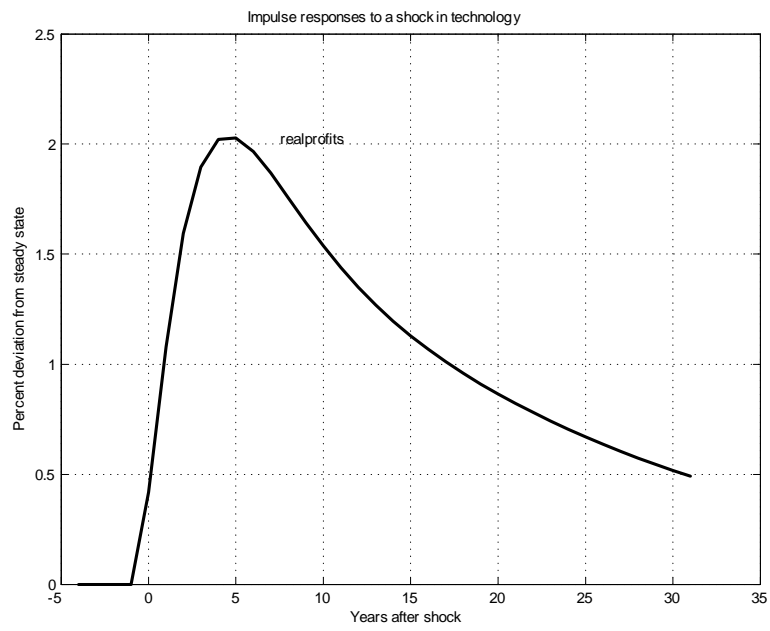
7.4 Figures

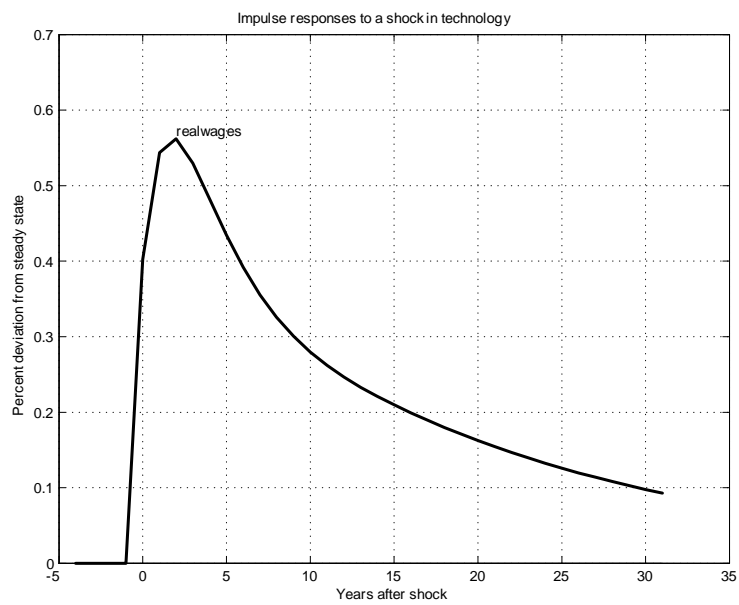
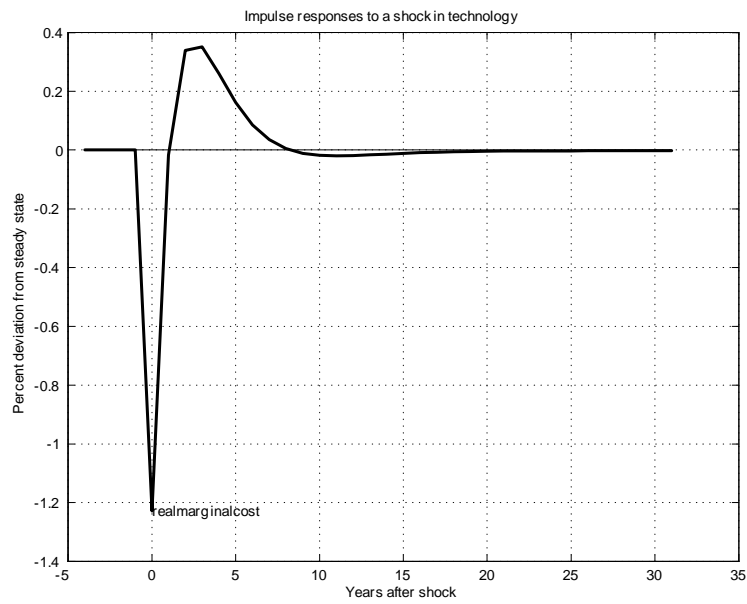
7.4.1 Impulse response functions to a shock in technology

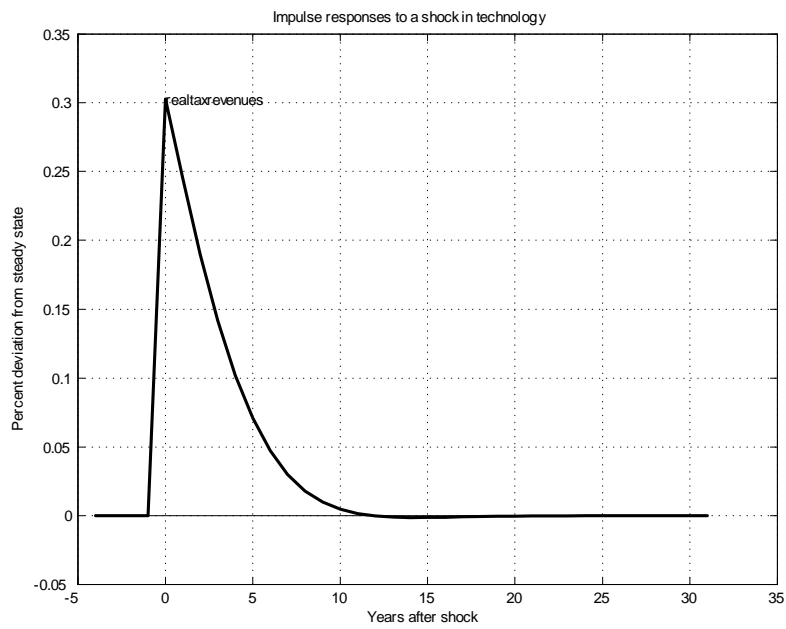
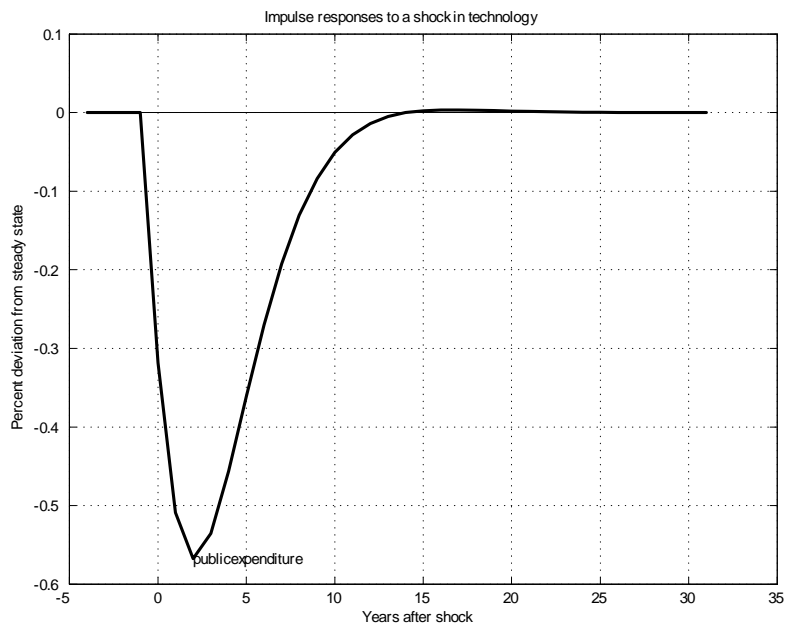












7.4.2 Impulse response functions to a shock in Taylor rule

