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## The effects of factor and sector biased technical change revisited

Robert Stehrer

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### Abstract

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In the trade-technology-wage debate, the effects of the various forms of technical progress on relative factor prices have been addressed in a number of contributions over the past decade. However, the existing literature is far from conclusive. The various contributions have either relied on specific assumptions, such as Leontief technologies or Cobb-Douglas demand, that have been decisive for the respective conclusions, or they used a more general framework, arriving at ambiguous results in many cases. In this paper we analyse a general equilibrium framework with CES production and CES demand functions, which allows for any discrete number of sectors and countries integrated via trade flows. Technologies are country- and sector-specific and endowment structures differ across countries. The necessary and sufficient conditions under which the relative wage rates are rising or falling in the domestic and foreign economies are derived. This is done for various types of factor- and sector-biased technical change taking place in a particular sector in either the home or foreign country. The conditions - depending on the relative skill intensity of the innovating sector, the elasticities of substitution in demand and supply, the relative factor endowment and the prevailing (equilibrium) relative wage rate - allow for straightforward economic interpretations. This permits to solve the cases classified as ambiguous in the existing literature and provides clear-cut conditions which are important for modelling and empirical research. Furthermore, the results are interpreted with respect to recent empirical studies where special emphasis is given to the sector-biased versus factor-biased hypothesis.

**JEL classification:** C62, C68, F16, O33;

**Keywords:** factor prices, technical change, trade, multisector and multicountry model;

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### The author

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*Robert Stehrer* is researcher at the Institute for International Economic Studies (wiiw). E-Mail: [stehrer@wiiw.ac.at](mailto:stehrer@wiiw.ac.at)



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# The effects of factor- and sector-biased technical change revisited

## 1 Introduction

The literature on the labour market effects of technical change in closed and open economies has grown substantially over the past decade. At the beginning the debate focused primarily on the impact of trade (mainly with developing countries) on the relative wage of skilled workers in the advanced countries (see e.g. Wood, 1997; Leamer, 1998). In the course of this debate, (mainly empirical) research (for an overview see Acemoglu, 2002b) soon centered on the relative importance of trade integration versus (factor- or sector-biased) technical progress in closed and open economies in explaining the rising wage differential between skilled and unskilled workers in the advanced economies, particularly the US. The importance of technical change is also emphasized in more recent empirical but also theoretical contributions, which focus in particular on the role of different types of technical progress, i.e. sector or factor bias of technical change. For instance, Haskel and Slaughter (2002) argue that it is mainly the sector bias of skill-biased technical change that matters, whereas e.g. Acemoglu (2002b) emphasizes the factor bias (skill bias) and explains the rising wage differential by an acceleration of this type of technical change. Of course, the issue of the impact of technology and technical change has a long tradition in the economic analysis of trade and welfare, dating back to Ricardo at least. But, as opposed to the endowment-based theories, that issue has gained momentum in the past decade (see e.g. Trefler, 1993). In the more recent literature the effects of changes in technology are discussed (see e.g. Dixit and Norman, 1980, for an overview). While these contributions address mainly the effects on trade and utility, they do not discuss the effects on relative factor prices - which is the main topic to be dealt with in this paper. The results of that literature are, however, not encouraging: the effects of technical change on welfare are found to be ambiguous for the home as well as for the foreign country and seemingly no general propositions can be derived (see also Findlay and Grubert, 1959). Similarly, the results from the debate of the effects of technical change on relative factor prices - to which we turn next in more detail - are, as already mentioned, not conclusive either.<sup>1</sup>

The theoretical results on the effects of technical change in closed and in (large) trading economies depend heavily on specific parameter assumptions in production and demand functions and the particular type of technical progress considered (e.g. factor-biased vs. sector-biased, local versus global, etc.). Haskel and Slaughter (2002) mention a number of studies and conclude that '... different studies have examined very different cases [and] general conclusions should not be made from any single study.' (p. 1765). Let us mention two of those studies which are relevant for this paper.<sup>2</sup> Recently, Xu (2001) analysed a  $2 \times 2 \times 2$ -model with different types of factor-augmenting (which is related to the Hicksian typology) technical change which clarifies this discussion to some extent. Although his results shed some light on the theoretical debate, some questions still remain unsolved. *First*, the effects analysed by Xu (2001) are in most cases ambiguous, particularly so for large trading economies. An even more detailed study seems thus to be necessary to further clarify the results. This is the main topic of the present paper. *Second*, from an empirical as well as modelling point of view it is important to see whether the analysis can be

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<sup>1</sup>Other recent contributions go beyond the trade versus technology question and also introduce, e.g., market structures (Neary, 2003), outsourcing (e.g. Feenstra and Hanson, 2001), firm size structures (e.g. Dinopolous et al., 2001), etc., which are not discussed here.

<sup>2</sup>One has to mention that both papers use a more general form of the production function as we will do.

extended beyond the two-sectors and two-countries framework. The model used below shows in which way this problem can be tackled. This is important as some catching-up countries do quite well in specific sectors (e.g. the skill-intensive sectors) in terms of productivity convergence, which has quite different implications on the relative wage rate in the advanced economies. Zhu and Trefler (2005) study a model with a continuum of goods where two seminal contributions (Dornbusch et al., 1977, 1980) are merged. This model includes factor endowment differences as well as technological differences between countries. However, in this contribution the restrictive case of a Cobb-Douglas demand structure is assumed which - as we shall argue below - may have specific implications. Further, it is also restricted to the case of just two trading economies where only a particular type of technical change in the 'South' (productivity growth is equal across all sectors) is analysed.

In the present paper we introduce a framework similar to that used in the latter contribution, but we apply it to a discrete number of sectors instead of a continuum of goods and extend it to allow for a discrete number of trading economies. It should be emphasized that the model allows simultaneously for differences in technologies, i.e. not only for differences in total factor productivity, as well as for differences in factor endowments, and thus goes beyond the standard frameworks based either on differences in productivity levels (Ricardo) or factor endowments (Heckscher-Ohlin).

In line with the contribution by Xu (2001) we analyse the effects of technical change on relative wages for various combinations of parameter values (especially assuming different elasticities of substitution in demand and in production) and different types of technical change.<sup>3</sup> For this model we shall derive exact criteria under which the skill premium will rise. These conditions depend on the skill intensity of the innovating sector, the elasticities of substitution in production and demand, the relative wage bill in the economy and the relative factor supply, and allow for an economic interpretation. In particular, the framework also allows to pinpoint the effects of technical change in specific sectors which are characterized by their relative skill intensity in both the advanced and less advanced economies. In this way the model allows to analyse the effects of different catching-up patterns of countries (see e.g. Stehrer and Woerz, 2003, on this).

The paper is structured as follows: In section 2 we introduce the model for a closed economy and derive the above-mentioned criteria. In this section we also discuss the types of technical change, which can be modelled using a CES production function. The framework is then extended to a multi-country model in section 3. Again, we derive criteria which allow to analyse the effects of the various forms of technical change taking place in the domestic or a foreign economy and study the effects on relative factor prices in the innovating and the other economies. We pay particular attention to those cases that are empirically relevant, i.e. skill-biased technical change and cases that provide a potential explanation for increasing relative wages. The final section concludes.

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<sup>3</sup>In this paper we focus exclusively on the effects of relative factor prices and do not analyse welfare effects as is common in the relevant literature. Preliminary numerical studies suggest that technical change has positive welfare effects on either total welfare or welfare of skills and labour in most cases; there are however some notable exceptions to this which deserve further research. The factors underlying these differences in welfare effects by skill types are the results on relative factor prices studied in this paper.

## 2 Technical change and relative factor prices in a closed economy

Let us now introduce the framework for a closed economy. Here we basically follow Zhu and Trefler (2005) but - instead of assuming a continuum of goods - we formulate the model for a discrete number of sectors  $i = 1, \dots, N$  which allows to generalize the latter contribution by introducing a CES demand structure. On the other hand, the results are comparable to those presented in Xu (2001) for the integrated economy but, first, allow for any discrete number of goods and, second, enable us to specify conditions for a rising relative wage rate in the cases that remain ambiguous in Xu (2001). We shall extend this framework to a discrete number of countries in section 3.

### 2.1 A multi-sectoral framework

#### 2.1.1 Consumers and producers

Consumers maximize a CES utility function denoted by  $U = \left( \sum_{i=1}^N \beta_i q_i^\varrho \right)^{1/\varrho}$  with  $0 < \beta_i < 1$ ,  $\sum_{i=1}^N \beta_i = 1$ , and  $-\infty < \varrho < 1$ . For  $\varrho \rightarrow 0$  the CES utility function collapses to a Cobb-Douglas utility function  $U = \prod_{i=1}^N q_i^{\beta_i}$ . Under the assumption that consumers maximize utility, the nominal expenditure shares for each good are given by

$$\gamma_i = p_i^{1-\varsigma} \beta_i^\varsigma / \sum_{j=1}^N p_j^{1-\varsigma} \beta_j^\varsigma \quad (2.1)$$

satisfying the constraints  $0 < \gamma_i < 1$  (for  $N > 1$ ) and  $\sum_i \gamma_i = 1$ . The parameter  $\varsigma := 1/(1 - \varrho)$  with  $0 < \varsigma < \infty$  denotes the elasticity of substitution between the good  $i = 1, \dots, N$ . For  $\varsigma \rightarrow 1$  (or equivalently  $\varrho \rightarrow 0$  in the utility function) the expenditure shares become constant  $\gamma_i = \beta_i$  as can be seen from equation (2.1).

Firms face a CES production function with two inputs skills,  $l_{iS}$ , and labour,  $l_{iL}$ , denoted by

$$q_i = A_i [\alpha_{iL} (a_{iL} l_{iL})^\rho + \alpha_{iS} (a_{iS} l_{iS})^\rho]^{1/\rho}$$

where  $l_{iz}$ ,  $z = L, S$  denote the amount of labour and skilled workers used in the production of good  $i$ , respectively.<sup>4</sup>  $A_i > 0$  denotes total factor productivity,  $a_{iz} > 0$  are the factor-augmenting parameters and we refer to  $0 < \alpha_{iz} < 1$  with  $\sum_z \alpha_{iz} = 1$  as share parameters.  $\rho$  determines the elasticity of substitution between the two factors with  $\sigma = 1/(1 - \rho)$  being the elasticity of production. The production function is homogenous of degree one (i.e. constant returns to scale). We shall refer to the parameters  $A_i, a_{iz}$  and  $\alpha_{iz}$  to distinguish between different types of technical change below. As commonly known, this CES function collapses for  $\rho \rightarrow 0$  to the Cobb-Douglas case (with elasticity of substitution equal to one)  $q_i = A_i (a_{iL} l_{iL})^{\alpha_{iL}} (a_{iS} l_{iS})^{\alpha_{iS}}$ ; for  $\rho \rightarrow -\infty$ , i.e. no factor substitution taking place, the production function is of the Leontief type  $q_i = A_i \min\{a_{iS} l_{iS}, a_{iL} l_{iL}\}$ . We have written this function in a general way, i.e. including both the parameters  $\alpha_{iz}$  and  $a_{iz}$  for reasons which will become clear below. The two factors receive a nominal wage rate of  $w_z$ ,  $z = L, S$ , respectively. We take the nominal wage rate of labour as *numéraire*, i.e.  $w_L = 1$ . The relative wage rate of the skilled workers is thus equal to their nominal wage rate as  $\omega := w_S/w_L = w_S$ . Using the nominal wage rate of labour as *numéraire*, i.e.  $w_L = 1$  the

<sup>4</sup>The usage of a CES production function is restrictive in some sense but it is easy to handle and straightforward conditions can be derived. The results may also hold for more general production functions (for which the elasticity of substitution is not constant) but then the results would have to be interpreted 'locally'.

relative wage can also be referred to as the 'skill premium'. Given the wage rates the cost-minimizing firms choose the amount of labour and skills per unit of output. Straightforward calculations yield that these input coefficients are given by

$$\tilde{a}_{iS} = \omega^{-\sigma} A_i^{-1} \alpha_{iS}^\sigma a_{iS}^{\sigma-1} B_i^{\sigma/(1-\sigma)} \quad (2.2a)$$

$$\tilde{a}_{iL} = A_i^{-1} \alpha_{iL}^\sigma a_{iL}^{\sigma-1} B_i^{\sigma/(1-\sigma)} \quad (2.2b)$$

with  $B_i := (\alpha_{iL}^\sigma a_{iL}^{\sigma-1} + \omega^{1-\sigma} \alpha_{iS}^\sigma a_{iS}^{\sigma-1})$ . Under the assumption of cost-minimization the ratio of skills to labour in industry  $i$ , i.e. the skill intensity, is given by

$$\tilde{a}_i := \frac{\tilde{a}_{iS}}{\tilde{a}_{iL}} = \omega^{-\sigma} \left( \frac{\alpha_{iS}}{\alpha_{iL}} \right)^\sigma \left( \frac{a_{iS}}{a_{iL}} \right)^{\sigma-1}$$

where  $\tilde{a}_{iz}$ ,  $z = S, L$  denotes the input of skills or labour per unit of output. The ratios of the cost shares of the factors in production of good  $i$ , i.e.  $\theta_{iz}$ , are then given by

$$\theta_i := \frac{\theta_{iS}}{\theta_{iL}} = \omega \tilde{a}_i = \omega^{1-\sigma} \left( \frac{\alpha_{iS}}{\alpha_{iL}} \right)^\sigma \left( \frac{a_{iS}}{a_{iL}} \right)^{\sigma-1}.$$

Under the non-profit condition prices equal unit costs, i.e.  $p_i = \tilde{a}_{iS} w_S + \tilde{a}_{iL} w_L$ . Inserting for the labour input coefficients  $\tilde{a}_{iz}$  and using  $w_S = \omega$  the price of good  $i$  is given by

$$p_i = A_i^{-1} B_i^{1/(1-\sigma)}. \quad (2.3)$$

This equation expresses prices as a function of technological parameters and the relative wage rate.

### 2.1.2 Typology of technical change

In the literature the most widely used definitions of technical progress are Hicks-neutral (or non-neutral, i.e. labour-using/saving or skill-using/saving) technical progress (see Hicks, 1932) and labour- versus skill-augmenting (dating back to Harrod, 1942). From a modelling point of view the factor-augmenting definition is most useful as this is related to a change in parameter values  $a_{iz}$ . However, for a proper interpretation of the results, the Hicksian definitions are more useful. Let us therefore briefly summarize these definitions for the CES production function used in this paper. Technical progress is factor-augmenting if it raises output in the same way as does an increase in the supply (use) of this factor. In general, the production function in this case takes the form  $Y = f(l_L, a_S(t)l_S)$  for skill-augmenting and  $Y = f(a_L(t)l_L, l_S)$  for labour-augmenting technical progress (see Harrod, 1942; Robinson, 1938; Uzawa, 1961). On the other hand, technical progress is classified as Hicks-neutral if the ratio of marginal products remains unchanged for a given factor input ratio (see Hicks, 1932). This corresponds to the case in which the factor intensity  $\tilde{a}_i$  remains constant at a fixed relative wage rate  $\omega$ . Thus, Hicks-neutral technical progress is of the form  $Y = A(t)f(l_L, l_S)$ . If the factor intensity remains not constant at a constant relative wage rate then technical progress is defined as either skill-saving (labour-using) or labour-saving (skill-using) if the skill intensity is falling or rising, respectively. As pointed out in Xu (2001) in a more general way there are some relationships between these definitions of technical progress. These are summarized with respect to the CES production function specified above in table 2.1. A change in total factor productivity  $A_i$  is Hicks-neutral irrespective of the elasticity of substitution between the two factors. In the Cobb-Douglas case, i.e.  $\sigma = 1$ , a change in total factor productivity  $A_i$  or a change in any of the factor-augmenting

Variable	$\sigma < 1$	$\sigma = 1$	$\sigma > 1$
$\Delta A_i > 0$	Hicks neutral	Hicks neutral	Hicks neutral
$\Delta a_{iS} > 0$	Labour using Skill augmenting	Hicks neutral	Skill using Skill augmenting
$\Delta a_{iL} > 0$	Skill using Labour augmenting	Hicks neutral	Labour using Labour augmenting
$\Delta \alpha_{iS} > 0$	Skill using	Skill using	Skill using
$\Delta \alpha_{iL} > 0$	Labour using	Labour using	Labour using

Table 2.1: Correspondences between types of technical progress

parameters  $a_{iz}$  is Hicks-neutral. As can be seen above, the input ratio  $\tilde{a}_i$  and cost share ratio  $\theta_i$  in this case depend only on the relative wage rate and the parameters  $\alpha_{iz}$ . Only a change in the share parameters  $\alpha_{iz}$  would imply a biased technical progress in the Hicksian sense as indicated in table 2.1.<sup>5</sup>

A change in the factor-augmenting parameters  $a_{iz}$ , however, must be interpreted more carefully when related to the Hicksian definitions of technical change. As can be seen from the expression for the skill intensity  $\tilde{a}_i$  an increase of the parameter  $a_{iS}$ , i.e. skill-augmenting technical change, is labour-using if  $\sigma < 1$ , Hicks-neutral if  $\sigma = 1$  (as already mentioned) and skill-using if  $\sigma > 1$ , i.e. the Hicksian classification depends on the elasticity of substitution. When  $\sigma < 1$  the elasticity of substitution is low (in the extreme case the two factors become perfect complements) which explains why in the presence of skill-augmenting technical change there is higher relative demand for labour. On the other hand, if  $\sigma > 1$  skills and labour are more substitutable and skill-augmenting technical change leads a relative increase of skills. In analogy, this can be applied to an increase in the labour-augmenting parameter  $a_{iL}$ . Finally, a change in the intensity parameters  $\alpha_{iz}$  would always be factor-biased as indicated in table 2.1.<sup>6</sup> The ratio of the cost shares  $\theta_i$  at constant factor prices is constant if technical progress is Hicks-neutral, rising if technical progress is skill-using, and falling if technical progress is of the labour-using type. Acemoglu (2002b), who provides a similar analysis<sup>7</sup> notes that most empirical studies show that  $\sigma \geq 1$ . Further, there seems ample evidence that technical change was skill-biased in the last decades (see Acemoglu, 2002b, for an overview) which corresponds to an increase in  $a_{iS}$  and  $\sigma > 1$ .

Finally let us note that an increase of either  $a_{iS}$  or  $a_{iL}$  is cost-reducing in any case at constant factor prices which allows to assume that the innovation is actually introduced. This is shown in appendix section A.1.

<sup>5</sup>This is one reason why we have explicitly included the parameters  $\alpha_{iz}$  in the production function; else these parameters would implicitly assumed to be  $\alpha_{iz} = 0.5$ .

<sup>6</sup>In this context, it is worth noting that care has to be taken when comparing different contributions of the recent literature. For example, Haskel and Slaughter (2002) use a production function which - in our notation - sets  $a_{iz} = 1$  and thus considers only changes in the skill intensity parameters  $\alpha_{iz}$ . On the other hand, Xu (2001), Zhu and Trefler (2005), and Davis (1998) use a functional form in which only the factor-augmenting parameters  $a_{iz}$  are included.

<sup>7</sup>In this paper, however, the demand side effects of a change in technological parameters are not taken into account and the analysis of the effects of technical change is restricted to changes in the production function and relative endowment. Furthermore, the paper provides reasons to explain the skill-biased nature of technical change (see also Acemoglu, 2002a, on this).



### 2.1.3 Factor endowment and equilibrium

The economy is endowed with labour  $h_L$  and skills  $h_S$ ; the relative factor endowment of skills will be denoted by  $h := h_S/h_L$ . The factor market clearing conditions are  $h_z = \sum_{i=1}^N \tilde{a}_{iz}q_i$ . Using the full employment assumption total income  $Y = \omega h_S + h_L = h_L(\omega h + 1)$ .

The equilibrium relative wage rate  $\omega$  can then be derived in the following way. Calculating employment rates (i.e. labour demand over supply) for skills and labour and setting the difference to zero yields the equilibrium relative wage rate. These employment rates can be written as

$$\begin{aligned} E_S &= \frac{1}{h_S} \sum_{i=1}^N l_{iS} = \frac{1}{h_S} \sum_{i=1}^N q_i \tilde{a}_{iS} = \frac{Y}{h_S} \sum_{i=1}^N \frac{\gamma_i}{p_i} \tilde{a}_{iS} \\ E_L &= \frac{1}{h_L} \sum_{i=1}^N l_{iL} = \frac{1}{h_L} \sum_{i=1}^N q_i \tilde{a}_{iL} = \frac{Y}{h_S} h \sum_{i=1}^N \frac{\gamma_i}{p_i} \tilde{a}_{iL}. \end{aligned}$$

Inserting for  $\gamma_i, p_i, \tilde{a}_{iz}$  from equations (2.1), (2.3) and (2.2) expresses them as functions of preference parameters, technological parameters and the relative wage rate. In equilibrium the condition  $E(\omega) := E_S(\omega) - E_L(\omega) = 0$  has to be satisfied. This implies that the employment rates are equal for labour and skills. Full employment in absolute terms for both factors is then assured by inserting  $\omega^*$  into the expression for  $Y = h_L(\omega h + 1)$ . As  $Y$  and  $h_S$  are positive numbers the equilibrium condition can be rewritten as

$$\sum_{i=1}^N \frac{\gamma_i}{p_i} \tilde{a}_{iS} = h \sum_{i=1}^N \frac{\gamma_i}{p_i} \tilde{a}_{iL}. \quad (2.4)$$

It can be shown that a unique and Walrasian stable equilibrium always exists if  $\sigma > 0$ . For a Leontief technology (i.e.  $\sigma = 0$ ) such an equilibrium may not exist at all. Rewriting the equilibrium condition yields  $\sum_{i=1}^N \frac{\gamma_i}{p_i} \tilde{a}_{iS} / \sum_{i=1}^N \frac{\gamma_i}{p_i} \tilde{a}_{iL} = h$ . The lhs of this equality may be referred to as *relative demand curve*  $D$  and the rhs as the *relative supply curve*  $S$ , respectively. The latter would be a vertical in a diagram with the relative wage on the vertical and the relative demand and supply at the horizontal axis. Such relative supply and demand curves are used e.g. in Wood (1997) for analysing the employment effects of shifts in trade patterns, and in Acemoglu (2002b) to analyse the effects of skill-biased technical change. For further exposition we shall however use equation 2.4 and shall refer to the lhs simply as DEMAND CURVE and to the rhs as SUPPLY CURVE. It can be shown that the demand curve is strongly monotonically downward sloping with  $\omega$ . This reflects substitution processes in production and demand, i.e. a higher relative wage rates of skills implies that relatively less skills are demanded as this factor and the relatively more skill-intensive goods becomes more expensive. The supply curve is upward sloping for a wide range of parameters. However, it may become vertical or even downward sloping as well depending on the specific values of the elasticity of substitution in production and demand. In this case it can be shown that  $\frac{dD}{d\omega} < \frac{dS}{d\omega}$ , which implies Walrasian stability also in this case. Note that in all cases the RELATIVE demand curve as described above is thus downward sloping.

Changes in technological parameters shift (rotate) the curves right or left (clockwise or counterclockwise). It is clear that a simultaneous shift of the supply curve to the left and of the demand curve to the right implies an increase in the equilibrium relative wage rate (and a decrease in the other case). This situation is graphically presented in panel 1 of figure 2.1. If both curves are shifting into the same direction due to a change in a technological parameter, the effect on the relative wage rate becomes ambiguous. Assume that both curves shift to the right: The equilibrium wage rate rises if the shift of

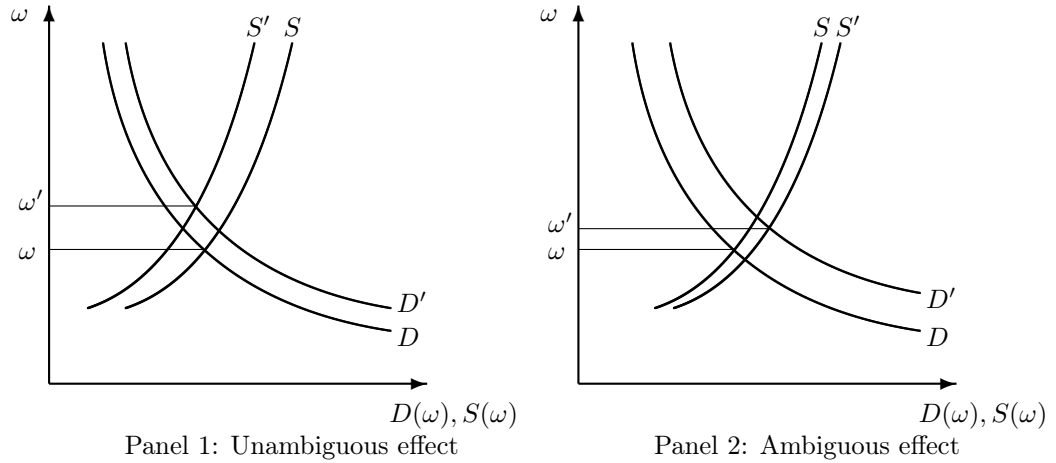


Figure 2.1: Relative demand and supply curves

the demand curve is - at the initial equilibrium wage rate - larger than the shift of the supply curve (this situation is shown in panel 2 of figure 2.1). Formally this can be expressed as  $\frac{\partial D}{\partial X} > \frac{\partial S}{\partial X}$  where  $X$  denotes any technological parameter  $A_i, a_{iz}$  or  $\alpha_{iz}$ . If both curves shift to the left, the supply curve must change relatively more (in absolute terms) compared to the demand curve to ensure a rising relative wage rate. As both derivatives with respect to the parameter  $X$  are negative in this case, the condition is again  $\frac{\partial D}{\partial X} > \frac{\partial S}{\partial X}$ . We shall use this inequality to analyse the effects of changes in technological parameters on the relative wage rate. Finally, the equilibrium condition (2.4) shows that a change in the size of the economy which keeps the relative endowment  $h$  constant has no effect on the wage rate; a rising relative endowment of skills shifts the supply curve to the right whereas the demand curve remains stable, which implies a declining relative wage rate.

## 2.2 The effects of factor- and sector-biased technical change

Inserting for  $\gamma_i, \tilde{a}_{iz}, p_i$ , from equations (2.1), (2.2), and (2.3) in equation (2.4) expresses the equilibrium condition as a function of preference parameters, technological parameters, the relative wage rate, and relative factor endowment, i.e.

$$\omega^{-\sigma} \sum_i \beta_i^\zeta A_i^{\zeta-1} \alpha_{iS}^\sigma a_{iS}^{\sigma-1} B_i^{\frac{\sigma-\zeta}{1-\sigma}} = h \sum_i \beta_i^\zeta A_i^{\zeta-1} \alpha_{iL}^\sigma a_{iL}^{\sigma-1} B_i^{\frac{\sigma-\zeta}{1-\sigma}} \quad (2.5)$$

with  $B_i := (a_{iL}^{\sigma-1} \alpha_{iL}^\sigma + \omega^{1-\sigma} a_{iS}^{\sigma-1} \alpha_{iS}^\sigma)$ ;  $B_i^{1/(1-\sigma)}$  has an interpretation as price index for input factors.<sup>8</sup>

Let us now start with a detailed analysis of the effects of an increase in total factor productivity  $A_k$  and an increase of the skill-augmenting parameter  $a_{kS}$  in a specific sector  $k$  on the relative wage rate  $\omega$ ; we shall denote this type of technical change by SATC.<sup>9</sup> In the following it is essential to distinguish between skill- and labour-intensive sectors. As the number of sectors can be larger than the number of factors, the

<sup>8</sup>One may also replace the elasticity of substitution in production  $\sigma$  by a sector-specific elasticity of substitution  $\sigma_i$ . For the sake of notational convenience we do not make use of this.

<sup>9</sup>The same analysis can be applied to changes in the labour-augmenting parameter  $a_{kL}$  but this shall not be discussed here as the results are analogous when applying the Hicksian definitions of technical change.

factor intensities are no longer a function of technological properties only, but become dependent upon equilibrium relationships (see e.g. Dixit and Norman, 1980). We thus introduce the following terminology:

**DEFINITION OF SKILL-INTENSIVE SECTORS:** We define a sector to be skill-intensive if  $\theta_k > \omega^*h$  and to be labour-intensive if  $\theta_k < \omega^*h$  where  $\omega^*$  denotes the relative wage rate in equilibrium. This can also be expressed as  $\tilde{a}_i > h$  or  $\tilde{a}_i < h$  by dividing both sides with  $\omega^*$ ; i.e. a skill-intensive sector has a higher skill ratio compared to the relative endowment  $h$  of the economy.

### 2.2.1 Sector-biased technical change

Let us first discuss the effect of an increase in total factor productivity in a particular sector  $k$ . Calculating<sup>10</sup> the first derivatives with respect to  $A_k$ , using the condition which has to be satisfied for the relative wage rate to rise,  $\frac{\partial D}{\partial A_k} > \frac{\partial S}{\partial A_k}$ , and simplifying yields  $(\varsigma - 1)\theta_k > wh(\varsigma - 1)$  or, stated differently,

$$\begin{aligned} \theta_k < \omega h & \quad \text{if } \varsigma < 1 \\ \theta_k > \omega h & \quad \text{if } \varsigma > 1. \end{aligned}$$

For  $\varsigma = 1$  (i.e. Cobb-Douglas demand) total factor productivity vanishes from the equilibrium condition (2.5) which implies that a change in total factor productivity has no effect whatsoever on the relative wage rate. The effect only depends on the elasticity of substitution of demand  $\varsigma$  and on whether the sector is skill- or labour-intensive according to the definition above. Let us distinguish the two cases.

**FIRST CASE WITH  $\varsigma < 1$ :** The relative wage rate will rise if the innovating sector is labour-intensive according to the definition above. As the price of this particular good is falling, demand for this good is rising whereas demand for other goods is falling as a consequence of the substitution effect; however, as  $\varsigma < 1$  the income effect is larger as compared to the substitution effect, which implies that demand for all other goods is rising as well. As this includes a number of goods with higher skill intensities as compared to the innovating sector - which is labour-intensive - this implies a rising demand for skills in total. The opposing forces to a rising skill premium are that, first, demand for the innovating sector is rising relative to the other sectors by the substitution effect; second, total factor productivity growth in a labour-intensive sector implies that less labour is used and thus the skill premium tends to rise; and third, at the new equilibrium each sector has become less skill-intensive as the relative wage is higher. These forces are not strong enough to prevent a rising wage rate.

**SECOND CASE WITH  $\varsigma > 1$ :** In this case the inequality sign reverses and the relative wage rate rises if the innovating sector is skill-intensive according to our definition. Demand in the innovating sector increases due to the substitution effect; although the income effect remains positive for all sectors the substitution effect dominates and thus demand for all other goods is decreasing. As now the number of relatively more labour-intensive goods is large, relative demand for skills is increasing. This effect dominates the opposing forces, i.e. an increase in total factor productivity in a skill-intensive sector and that each sector has become less skill-intensive in the new equilibrium.

The second case is in line with the findings of Haskel and Slaughter (2002) arguing that the reduction of the skill premium was driven by a concentration of technical change in the labour-intensive sectors in the 1970s; conversely, the rise in the skill premium in the 1980s and 1990s may be explained by concentration

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<sup>10</sup>Henceforth,  $\omega$  denotes the relative wage rate in equilibrium.

of technical change in the skill-intensive sectors.<sup>11</sup> With respect to catching-up economies there is some evidence that the productivity growth rates are higher in the skill-intensive (high-tech) sectors as the initial gap is larger and the speed of catching-up is faster in these sectors (see e.g. Landesmann and Stehrer, 2001, for evidence on labour productivity). This pattern of catching-up would lead to an increasing wage premium in the successfully catching-up countries such as the East Asian countries or Mexico and some of the Central and Eastern European countries. Further below we shall find out in which way this result still holds when taking account of trade and international specialization (see section 3). Let us now turn to the effects when allowing for factor-biased technical change.

### 2.2.2 Sector- and factor-biased technical change

We discuss the effects of a change in the skill-augmenting parameter in a particular sector  $k$ . Here we shall mainly focus on the effects of skill-using technical change, i.e. a rise in the skill-augmenting parameter  $a_{kS}$  under the assumption  $\sigma > 1$  (see table 2.1 above), as this seems to be the most relevant case empirically (see e.g. Haskel and Slaughter, 2002; Acemoglu, 2002b).<sup>12</sup> The condition under which the relative wage rate will rise is derived in the appendix and given by<sup>13</sup>

$$(\sigma - 1) + (\zeta - 1)\theta_k > \omega h(\zeta - \sigma). \quad (2.6)$$

Let us define the critical value of the cost ratio as  $\theta_* := \omega h \frac{\zeta - \sigma}{\zeta - 1} - \frac{\sigma - 1}{\zeta - 1}$ . This is a function of the elasticities of substitution  $\sigma$  and  $\zeta$  and the relative wage bill  $\omega h$  and will play a vital role in the interpretation of the results. For later use we note that the first derivative with respect to  $\zeta$  is negative for  $\sigma < 1$ , zero if  $\sigma = 1$  and positive for  $\sigma > 1$ . The derivative with respect to  $\sigma$  is negative for  $\zeta > 1$  and positive for  $\zeta < 1$ . Figure 2.2 sketches the graphs for different combinations of  $\sigma$  and  $\zeta$  where we have to distinguish the cases  $0 < \zeta < 1$ ,  $\zeta = 1$  (see below), and  $\zeta > 1$ .  $\theta_*$  as a function of  $\omega h$  passes through  $(-1, -1)$  for all parameter values  $\zeta \neq 1$  and  $\sigma$ . Let us discuss the various cases in detail.

**FIRST CASE: LOW ELASTICITY OF SUBSTITUTION IN DEMAND** Let us again start with the assumption that goods are complements, i.e.  $0 \leq \zeta < 1$ . In this case SATC leads to an increase in the relative wage rate if the skill intensity is lower than the critical value  $\theta_*$ . This can also be seen from condition (2.6) above, which can be written as  $\theta_k < \omega h \frac{\zeta - \sigma}{\zeta - 1} - \frac{\sigma - 1}{\zeta - 1}$  (the inequality sign reverses as  $\zeta - 1 < 0$ ).

- (1) For  $\sigma \leq \zeta$  the critical value  $\theta_*$  becomes negative and thus the condition can never be satisfied as  $\theta_k > 0$  by definition. The relative wage rate is decreasing irrespective of in which sector SATC takes place. In particular this implies that under the assumption of a Leontief technology, i.e.  $\sigma = 0$ , and  $\zeta < 1$  SATC leads to a decrease in the relative wage in any case. The reason for this is the strong skill-saving (labour-using) character in the Hicksian sense.
- (2) If  $\sigma$  is larger than  $\zeta$  (but still remains less than one) the effect on the relative wage rate becomes ambiguous. As indicated in panel 1 of figure 2.2 the critical value may become positive if  $\omega h$  is

<sup>11</sup>Haskel and Slaughter (2002) analyse skill-biased technical change in open economies. The result above shows that even unbiased technical change in a closed economy could explain the movements of relative wages.

<sup>12</sup>Here we investigate the case of a rise in the skill-augmenting parameters. A similar analysis can be undertaken for a rise in the labour-augmenting parameter,  $a_{kL}$ , which would be skill-using if  $\sigma < 1$ . The condition in this case would become  $(\sigma - 1) + (\zeta - 1)\theta_k^{-1} < (\zeta - \sigma)(\omega h)^{-1}$  for which an analogous interpretation applies.

<sup>13</sup>When assuming sector-specific elasticities of substitution in production the condition would become  $(\sigma_k - 1) + (\zeta - 1)\theta_k > \omega h(\zeta - \sigma_k)$ .

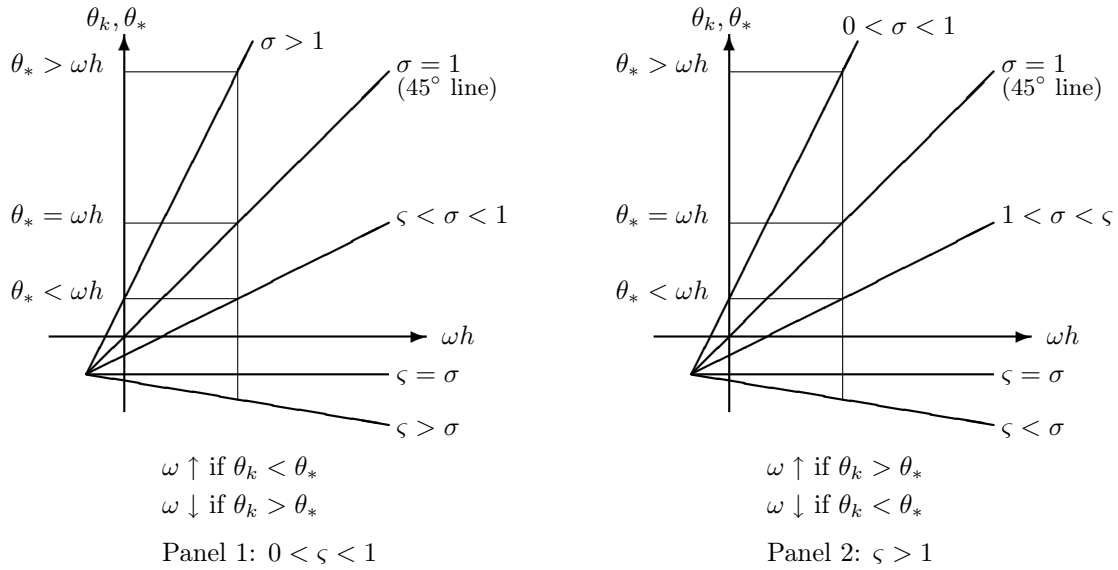


Figure 2.2: Critical values

high enough. In this case it may happen that SATC taking place in the most labour-intensive industries (i.e. industries characterized by  $\theta_k \in ]0; \theta_*[$  if  $\theta_* > 0$ ) raises the relative wage rate; SATC in industries characterized by  $\theta_k > \theta_*$  leads to a decrease in  $\omega$ . This critical value  $\theta_*$  will however always be lower than  $\omega h$ . Thus the range of sectors for which SATC induces a decrease in  $\omega$  includes labour-intensive industries (with the exception of the most labour-intensive ones) as well as all skill-intensive industries (i.e. with a cost ratio larger than  $\omega h$ ). Thus, the ambiguous results in Xu (2001) are explained by taking the skill intensity of the innovating sector into account. In a two-sector model the difference of the skill intensities in both sectors leads either to a positive or a negative effect. Let us discuss the case  $\theta_k < \theta_*$  in more detail. Due to the labour-using character of SATC, demand for labour is rising (at constant factor prices); the effect is less strong compared to a smaller  $\sigma$  as discussed in the previous paragraph. On the other hand - as the elasticity of substitution in goods demand is low - demand for goods in all other sectors is rising as the income effect dominates the substitution effect. As these sectors are relatively more skill-intensive compared to the innovating sector, demand for skills is increasing. As this range is large enough - the innovating sector is 'very' labour-intensive - this effect dominates the first and thus relative demand for skills and the relative wage rate are rising. If the innovating sector is characterized by a skill intensity above  $\theta_*$ , the labour-using effect is less pronounced as this sector is more skill-intensive. Further, demand for goods for a larger range of more labour-intensive sectors is increasing, which induces a reduction in the relative wage rate.

For a lower  $\sigma$  the range of labour-intensive sectors below the critical value becomes smaller as the labour-using character is more pronounced. An even larger range of relatively more skill-intensive sectors is needed such that the relative wage rate rises for a fixed  $\zeta$ . If  $\sigma \leq \zeta$  we have already seen above that the relative wage is decreasing in any case. Second, a larger parameter value  $\zeta$  (still

lower than one) also implies a smaller range  $]0; \theta_*[$  as the substitution effect for goods is higher and the shift of demand towards the innovating sectors is more pronounced. Thus, an even larger number of relatively more skill-intensive sectors is needed to induce an increase in the relative wage rate.

- (3) For  $\sigma = 1$  (Cobb-Douglas production) the graph of the critical value becomes a 45° line.<sup>14</sup> A change in a factor-augmenting parameter  $a_{kz}$  is Hicks-neutral. Qualitatively, this is exactly the same case as already discussed above for a change in  $A_k$  (see section 2.2.1). In this case only the sector bias of technical change matters.
- (4) Finally, if  $\sigma > 1$  SATC is skill-using (labour-saving) in the Hicksian sense, which is the empirically relevant case (see e.g. Haskel and Slaughter, 2002). The critical value now is always above the average value, i.e.  $\theta_* > \omega h$ , as can be seen in figure 2.2. SATC raises the relative wage rate apart from cases where the innovating sector is 'very' skill-intensive (in the sense that  $\theta_k > \theta_*$ ). The range of sectors for which  $\omega$  is rising includes all labour-intensive industries as well as those characterized by higher than average skill intensities. As SATC is skill-using in the Hicksian sense this implies rising demand for skills in the innovating sector. If it takes place in a sector with skill intensity below the critical value  $\theta_*$ , demand for goods in all other sectors will rise as well (as we still have  $\zeta < 1$ , the income effect dominates the substitution effect). If the innovating sector is not too skill-intensive the net effect is an increase in demand for skills and hence a rising relative wage. If, however, innovation takes place in a sector characterized by  $\theta_k > \theta_*$ , demand for goods in a wide range of relatively more labour-intensive sectors is rising, which explains the reduction in the relative wage rate.

Let us again discuss this result with respect to the findings in the literature. Haskel and Slaughter (2002) argue that skill-biased technical change was concentrated in the labour-intensive sectors in the 1970s and in the skill-intensive sectors in the 1980s and 1990s. Given this result, technical change would have had to be concentrated in the 'very' skill-intensive sectors in the 1970s to explain the decrease of relative wages; however, this seems to be at odds with the empirical facts. On the other hand, if technical change was concentrated in all sectors with the exception of the 'very' skill-intensive ones the rising relative wage rate in the 1980s and 1990s could be explained. For less advanced economies, which are performing relatively better in the labour-intensive sectors and some of the skill-intensive sectors, skill-using technical change is thus a potential explanation for rising relative wages. For this one has only to exclude that technical change occurs mainly in the most skill-intensive sectors of the catching-up economies. These two interpretations depend, however, on the assumption of  $\zeta < 1$ . In the next sections we analyse the cases  $\zeta = 1$  and  $\zeta > 1$ .

Summarizing, the ambiguous results in Xu (2001) may thus be explained by taking the (relative) skill intensity of the innovating sector into account: If  $\sigma < 1$ , the effect will always be negative if technical change takes place in a skill-intensive sector; if technical change occurs in a labour-intensive sector the relative wage rate may rise if this is a 'very' labour-intensive sector, otherwise it will fall. For  $\sigma > 1$  and  $\zeta < 1$ , SATC in a 'very' skill-intensive sector leads to a decrease of the relative wage; the effect is positive if the sectors is below the critical value  $\theta_*$ . If technical change takes place in a labour-intensive sector

<sup>14</sup>The condition cannot be derived directly from the equilibrium equation as this would involve a division by zero. However, one can show easily that in this case the condition becomes  $(\zeta - 1)\theta_k > \omega h(\zeta - 1)$ .

the effect will be unambiguously positive. Let us compare these results also to the results of growth in total factor productivity. We have seen that under the assumption  $\varsigma < 1$  relative wages are rising if neutral technical change takes place in the labour-intensive sectors. If technical change is of the skill-using character the range of sectors where the effect on  $\omega$  is positive becomes even larger.

**SECOND CASE: COBB-DOUGLAS DEMAND** If  $\varsigma = 1$  the condition becomes  $(\sigma - 1) > \omega h(1 - \sigma)$  or  $(\sigma - 1) > -\omega h(\sigma - 1)$ . Again one has to distinguish several cases:

- (1) Under the assumption  $0 \leq \sigma < 1$  the condition reduces to  $1 < -\omega h$  which can never be satisfied as  $\omega h > 0$  by definition; thus, if factors are complements, SATC unambiguously reduces  $\omega$  as in this case SATC is skill-saving. As can be seen from the derivatives of the supply and demand curves (see appendix A.2) the demand curve shifts to the left whereas the supply curve shifts to the right.
- (2) The equilibrium condition can be solved explicitly if  $\sigma = 1$  and becomes  $\omega = \frac{1}{h} \frac{\sum_{i=1}^N \beta_i \alpha_{iS}}{\sum_{i=1}^N \beta_i \alpha_{iL}}$ . Neither a change in total factor productivity nor a change in factor-augmenting parameters (which are Hicks-neutral) would affect the equilibrium wage rate.
- (3) In the third case,  $\sigma > 1$ , the condition simplifies to  $1 > -\omega h$ , which is satisfied in any case. Hence, if factors are substitutes, SATC, which is skill-using now, leads to a higher relative wage rate irrespective of the skill intensities of the innovating sector.

Summarizing, if  $\varsigma = 1$  it is only the factor bias and not the sector bias of technical change that matters. The effect on  $\omega$  solely depends on the elasticity of substitution in production and the effects can easily be classified by using the Hicksian definition of technical change. The result by Krugman (2000) that only the factor bias matters thus depends heavily on the assumption of Cobb-Douglas preferences; the direction of the effects depends on the elasticities of substitution in production.<sup>15</sup>

**THIRD CASE: HIGH ELASTICITY OF SUBSTITUTION IN DEMAND** Let us now consider the case  $\varsigma > 1$ , which is a frequently used assumption. Now an increase in the skill-augmenting parameter taking place in a sector with skill intensity larger than the critical value  $\theta_*$  leads to an increase in the relative wage rate. As the condition (2.6) becomes  $\theta_k > \omega h \frac{\varsigma - \sigma}{\varsigma - 1} - \frac{\sigma - 1}{\varsigma - 1}$  the inequality sign is not reversed as  $\varsigma - 1 > 0$ . Again, let us discuss the cases with respect to different values of  $\sigma$ ; these are graphically shown in panel 2 of figure 2.2.

- (1) If  $0 \leq \sigma < 1$  the critical value is  $\theta_* > \omega h$  and SATC will lead to a higher relative wage rate only when taking place in the most skill-intensive sectors. Although SATC is skill-saving, the fact that  $\varsigma > 1$ , i.e. the substitution effect dominates the income effect, implies that demand shifts to this sector whereas it is falling for all other ones (there is a wide range of less skill-intensive sectors) and as the skill intensity of the innovating sector is relatively high the relative wage is rising. On the other hand, if innovation takes place in a sector characterized by  $\theta_k < \theta_*$  demand in a larger range of (relatively) skill intensive sectors is falling whereas a more labour-intensive sector as compared to

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<sup>15</sup>Krugman (2000) assumed Leontief technologies. To analyse the effects of skill-using technical change one would thus have to assume an increase in the labour-augmenting parameter  $a_{kL}$ . But the same results would hold as well. Furthermore, the results that it is only the factor bias which matters, also arises in the cases assuming  $\varsigma \neq 1$  above if  $\sigma$  becomes either sufficiently large or sufficiently low (e.g.  $\sigma \leq \varsigma$  or  $\sigma \geq \varsigma$ , respectively).

above is attracting demand; the net effect is a falling relative wage rate (note that technical change in this case is labour-using).

The critical value is smaller the higher is  $\varsigma$ . The reason is that the substitution effect for goods demand increases such that even a lower skill intensity of the innovating sector suffices to induce a rising relative wage rate. On the other hand, the critical value is lower for a higher parameter value of  $\sigma$  as the skill-saving character is less pronounced. Note that the range of sectors for which SATC leads to a decreasing relative wage contains all labour-intensive sectors but also those with above-average skill intensity - which explains the ambiguity of the results in Xu (2001).

- (2) For  $\sigma = 1$  the graph of the critical value is again a 45° line. In this case we thus have  $\theta_* = \omega h$ . Again, this is the case already discussed above as technical change is Hicks-neutral.
- (3) Under the assumption  $1 < \sigma < \varsigma$ , SATC becomes skill-using, which is the empirically relevant case. The critical value may be positive (if  $\omega h$  is large enough). The relative wage is increasing for a large range of industries, but there may be some 'very' labour-intensive industries (i.e. industries with  $\theta_k < \omega h$ ) with SATC leading to a decreasing relative wage rate. If SATC takes place in a sector with skill intensity above the critical value, the same arguments as above apply; additionally technical change is skill-using, which explains that the critical value is lower. If SATC takes place in a 'very' labour-intensive sector, demand for a large range of skill-intensive sectors is falling whereas demand for the very labour-intensive good is rising. This effect dominates the other ones and  $\omega$  is falling. In this case a higher elasticity of substitution in demand  $\varsigma$  increases the critical value whereas a higher elasticity of substitution in factor demand lowers the critical value.
- (4) For  $\sigma \geq \varsigma$  the critical value  $\theta_k$  is always negative and thus the sector bias of SATC plays no role. The condition above is always satisfied and the relative wage rate is increasing in any case. The reason for this is the strong skill-using character. The sector bias of technical change would not play any role if this assumption is satisfied.

Again, these results are in line with the findings reported in Xu (2001) but additionally provide explanations for the ambiguous cases reported therein similar to above. Let us finally mention that cases (3) and (4) just discussed are important as the underlying assumption is that technical change is skill-biased. In these two cases skill-using technical change leads to a rising relative wage rate apart from cases where innovation takes place in the most labour-intensive sectors only. These results thus may explain why a number of empirical studies point to the importance of the factor bias rather than the sector bias for an explanation of the rising skill premium in the 1980s and 1990s (when assuming that technical change was concentrated in the skill-intensive sectors). However, the falling skill premium in the 1970s could only be explained by skill-biased technical change in the 'very' labour-intensive sectors. With respect to catching-up economies this would also provide an explanation for a rising wage differential if these countries are performing relatively better in the skill-intensive sectors (or, more exactly, not only in the most labour-intensive sectors) as proposed by Landesmann and Stehrer (2001). However, so far we have excluded effects of international specialization, which will be discussed in detail in the next section.

This concludes the detailed discussion of the effects of technical change on the relative wage rate in a closed economy.<sup>16</sup> We have shown in which way both the factor and the sector bias of technical

<sup>16</sup>Let us finally mention the results of a different modelling of technical change, namely, a change in the share parameters  $\alpha_{kz}$  as e.g. used in Haskel and Slaughter (2002). Since an increase of  $\alpha_{kS}$  is skill-using in any case, the elasticity of



change are important in providing a proper explanation of the effects of technical change. For skill-biased technical change (which is the empirically relevant case) it is the sector bias which matters when taking the effects on goods prices and demand into account; however, the direction of change depends crucially on the elasticity of substitution in demand. The results are summarized in table 2.2.

### 3 Effects of technical change in trading economies

So far we only dealt with the case of a closed economy. Now we extend this framework to a discrete number of trading economies, denoted by superscripts  $c = 1, \dots, C$ . If the number of sectors is larger than the number of factors, i.e.  $N > H$ , it is generally not possible to predict the patterns of trade. This problem will be dealt with by assuming a specific demand system which, basically, assumes that goods of a particular sector  $i$  are differentiated by country. We shall use Dixit-Stiglitz preferences and allow for different elasticities of substitution in, first, demand for particular goods  $i$  across countries denoted by  $\varphi$  (which is assumed to be larger than one)<sup>17</sup> and, second, in demand across goods (again denoted by  $\varsigma$ ). This in particular implies that consumers demand all goods available, i.e. from the domestic and all foreign countries; this may be interpreted as a love-for-variety for country-specific brands.<sup>18</sup> Consequently, the law of one price does not hold as each good is considered a specific brand. As each country produces and demands in all sectors, countries with a lower price capture a larger share of world production (when accounting for the size of the economies) which thus allows for specialization patterns. Furthermore, the model thus does not allow for complete specialization, and intra-industry trade (i.e. a country is importing and exporting in the same sector) will take place.

These specific assumptions on the demand structure imply that the results are no longer strictly comparable to the contributions by Xu (2001) and Zhu and Trefler (2005) (which basically require the law of one price to hold.) Still, the model remains analytically tractable, which again results in specific conditions allowing for straightforward economic interpretations and useful insights for discussing the effects of technical change in a setup of many trading economies.

#### 3.1 The model for trading economies

For an extension to many trading economies we assume that goods are differentiated by countries. The nominal expenditure shares for demand of country  $r$  in country  $c$  for a particular product  $i$  are then given by  $\gamma_j^{rc} = \frac{(\epsilon^{tr} p_j^r)^{1-\varphi} (\delta^r)^\varphi (P_j^{(t)})^{1-\varsigma} \beta_j^\varsigma}{P_j^{(t)}} \frac{1}{P^{(t)}}$  which satisfies  $0 < \gamma_j^{cr} < 1$  and  $\sum_{j,r} \gamma_j^{cr} = 1$  for all countries  $c = 1, \dots, C$ . In this expression  $\epsilon^{tr}$  denotes the exchange rate which expresses prices in a common currency of country  $t$ ;  $\delta^r$  and  $\beta_j$  are preference parameters. In the appendix section A.3 we show how this is derived from a nested CES utility function where we allow for different elasticities of substitution

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substitution in production  $\sigma$  plays no role in the qualitative results. The results can be summarized as follows: An increase in  $\alpha_{kS}$  implies a higher relative wage rate if it is cost-reducing in most cases. Only for substantially low or substantially high parameter values of  $\varsigma$  the effect may become ambiguous. As a similar interpretation applies, we shall not go into detail.

<sup>17</sup>Again one may consider cases where this elasticity is replaced by sector-specific elasticity. In the conditions given the parameter  $\varphi$  would then have to be replaced by  $\varphi_k$ .

<sup>18</sup>Romalis (2004) integrates a many-country version of a Heckscher-Ohlin model with a continuum of goods with monopolistic competition and transport costs. Each country produces different varieties of a particular sector and countries using their abundant factors more intensively capture a larger share of production and trade. Here a Cobb-Douglas demand and Cobb-Douglas production structure which is equal across industries is assumed.

$\Delta A_k > 0$	EoS in production		
EoS in demand	$\sigma = 0$	$0 < \sigma < 1$	$\sigma = 1$
$0 < \varsigma < 1$	+ if $\theta_k < \omega h$ - if $\theta_k > \omega h$	+ if $\theta_k < \omega h$ - if $\theta_k > \omega h$	+ if $\theta_k < \omega h$ - if $\theta_k > \omega h$
$\varsigma = 1$	0	0	0
$\varsigma > 1$	+ if $\theta_k > \omega h$ - if $\theta_k < \omega h$	+ if $\theta_k > \omega h$ - if $\theta_k < \omega h$	+ if $\theta_k > \omega h$ - if $\theta_k < \omega h$
$\Delta a_{iS} > 0$	EoS in production		
EoS in demand	$\sigma = 0$	$0 < \sigma < 1$	$\sigma = 1$
$0 < \varsigma < 1$	-	- if $\sigma \leq \varsigma$ - if $\sigma > \varsigma$ and $\theta_k > \theta_*( < \omega h)$ + if $\sigma > \varsigma$ and $\theta_k < \theta_*( < \omega h)$	- if $\theta_k > \theta_*( > \omega h)$ + if $\theta_k < \theta_*( > \omega h)$
$\varsigma = 1$	-	-	+
$\varsigma > 1$	- if $\theta_k < \theta_*( > \omega h)$ + if $\theta_k > \theta_*( > \omega h)$	- if $\theta_k < \theta_*( > \omega h)$ + if $\theta_k > \theta_*( > \omega h)$	+ if $\sigma \geq \varsigma$ + if $\sigma < \varsigma$ and $\theta_k > \theta_*( < \omega h)$ - if $\sigma < \varsigma$ and $\theta_k < \theta_*( < \omega h)$

Table 2.2: Summary of results for integrated economy

in demand across goods  $\varsigma$  and for a particular good across countries  $\varphi \geq 1$ ; the latter condition means that goods of the same sector  $j$  produced in different countries are substitutes.  $P_j^{(t)}$  and  $P^{(t)}$  are price indices for the composite good  $j$  and for the overall economy, respectively.<sup>19</sup> For the special case  $\varphi = 1$  the expenditure shares become a constant  $\gamma_j^{rc} = (\beta_j^{\varsigma} / \sum_k \beta_k^{\varsigma}) \delta_j^r$ .

The employment rates must be modified to include demand from other countries as well and have to be reformulated as

$$\begin{aligned} E_S^r &= \frac{1}{h_S^r} \sum_{i=1}^N l_{iS}^r = \frac{1}{h_S^r} \sum_{i=1}^N q_i^r \tilde{a}_{iS}^r = \frac{1}{h_S^r} \sum_{c=1}^C \sum_{i=1}^N \frac{\gamma_i^{rc} (\epsilon^{rc} Y^c)}{p_i^r} \tilde{a}_{iS}^r \\ E_L^r &= \frac{1}{h_L^r} \sum_{i=1}^N l_{iL}^r = \frac{1}{h_L^r} \sum_{i=1}^N q_i^r \tilde{a}_{iL}^r = \frac{1}{h_S^r} h^r \sum_{c=1}^C \sum_{i=1}^N \frac{\gamma_i^{rc} (\epsilon^{rc} Y^c)}{p_i^r} \tilde{a}_{iL}^r. \end{aligned}$$

The technical parameters  $A_k^c, a_{kz}^c$  and  $\alpha_{kz}^c$  may vary across countries and industries; however, we again assume that the elasticity of substitution in production  $\sigma$  is equal for all industries and countries.<sup>20</sup> The internal equilibrium requires  $E^r = E_S^r - E_L^r = 0$  and thus the condition becomes

$$\sum_{i=1}^N \frac{\tilde{a}_{iS}^r}{p_i^r} \sum_{c=1}^C \gamma_i^{rc} \frac{\sum_{j=1}^N \gamma_j^{cr}}{\sum_{j=1}^N \gamma_j^{rc}} = h^r \sum_{i=1}^N \frac{\tilde{a}_{iL}^r}{p_i^r} \sum_{c=1}^C \gamma_i^{rc} \frac{\sum_{j=1}^N \gamma_j^{cr}}{\sum_{j=1}^N \gamma_j^{rc}} \quad (3.7)$$

where we inserted balance-of-payments equilibrium conditions for  $\epsilon^{rc} Y^c$ . For fixed exchange rates and constant relative wage rates in the other countries  $c$  the same analysis with respect to existence, uniqueness and stability of the (internal) equilibrium in country  $r$  similar to the case of the closed economy applies. In the following we are interested in studying the effects of technical change taking place in a specific sector and country on the relative wage rate in this or a foreign economy. For this we can use the same method as above, i.e. taking the first derivative with respect to the technological parameter under consideration and evaluating this expression at fixed wage and exchange rates.

### 3.2 The effects of sector- and factor-biased technical progress in the domestic and foreign economy

As was done above, we shall again show the conditions for a rise in the relative wage and give an interpretation of them. The examples we discuss are, first, the effects of skill-biased technical change in an advanced economy on less advanced (developing) countries and, second, the effects of technological catching-up of less advanced economies on relative wages in the advanced economies. In particular we are interested in which cases one can explain the rising skill premium in the advanced as well as the less advanced (catching-up) economy (see Feenstra and Hanson, 2001). For analytical clarity we shall again assume that technical change takes place in a particular sector of either an advanced or a less advanced economy and study the effects on the innovating as well as on the other economy. The equilibrium condition when inserting for prices, input coefficients and expenditure shares is shown in the appendix section A.3. We start with the special case  $\varphi = 1$  already mentioned above.

<sup>19</sup>As preferences are assumed to be equal across countries, the country index does not appear in these expressions.

<sup>20</sup>Similarly to the closed economy case, one could introduce country- and sector-specific elasticities of substitution in production for analysing the effects of technical change. Again, we shall not do this for notational convenience.

### 3.2.1 A special case

We show in the appendix that the nominal expenditure shares are constant under the assumption of  $\varphi = 1$ . Denoting them by  $\beta_i^r$  the equilibrium condition can be written as

$$(\omega^r)^{-\sigma} \sum_{i=1}^N \beta_i^r (\alpha_{iS}^r)^\sigma (a_{iS}^r)^{\sigma-1} (B_i^r)^{-1} = h^r \sum_{i=1}^N \beta_i^r (\alpha_{iL}^r)^\sigma (a_{iL}^r)^{\sigma-1} (B_i^r)^{-1}$$

with  $B_i^r = (a_{iL}^r)^{\sigma-1} (\alpha_{iL}^r)^\sigma + (\omega^r)^{1-\sigma} (a_{iS}^r)^{\sigma-1} (\alpha_{iS}^r)^\sigma$ . One can see that this expression is similar to the expression in the case of a closed economy. As the expenditure shares are constant the exchange rates cancel out in this condition. Thus one can derive the internal equilibrium for each economy separately which determines the equilibrium wage rates  $\omega^c$  for  $c = 1, \dots, C$  and - given  $\omega^r$  and the parameters for all countries - one can easily calculate the equilibrium exchange rates to satisfy the external equilibrium condition (see footnote above).<sup>21</sup>

To analyse the effects of technological changes in the domestic economy the analysis is analogous to the case of a closed economy under the assumption  $\zeta = 1$  (see paragraph SECOND CASE above): First, a change in total factor productivity has no effect on the equilibrium relative wage; second, a higher skill-augmenting parameter  $a_{kS}^r$  has a negative effect on the relative wage rate if  $\sigma < 1$  and a positive effect if  $\sigma > 1$ . What are the effects of technological change in a foreign economy on the relative wage in the domestic economy? Here it is interesting to note that under the assumption of constant expenditure shares such changes have no effect on the domestic country  $r$  whatsoever.<sup>22</sup> This result however changes when allowing  $\varphi > 1$  to which we turn next.

### 3.2.2 Sector-biased technical change

The equilibrium condition is given in the appendix (see section A.3). In this section we analyse the effect of a change in total factor productivity from the viewpoint of economy  $r$ . We have to distinguish between technical change taking place in the domestic country  $r$  and a foreign country  $c \neq r$ . Let us discuss the latter case first.

TECHNICAL CHANGE IN A FOREIGN ECONOMY. Taking derivatives with respect to  $A_k^c$  gives the condition for the home country  $\theta_k^r < \omega^r h^r$  (see appendix). The effect on the equilibrium wage rate in economy  $r$  only depends on the cost ratio and the relative wage bill in this country, and not on that in the innovating (foreign) economy. Thus, the effect only depends on the ranking of the sector in the economy  $r$  under consideration and not on the characteristic of the sector in the innovating economy. For example, let us assume that total factor productivity is increasing in a particular sector  $k$  of a foreign country  $c$ . If this sector is labour-intensive in the domestic country  $r$ , i.e. satisfies  $\theta_k^r < \omega^r h^r$ , the relative wage rate will rise. As  $\varphi > 1$  relative more of this good is demanded in country  $c$  and less is demanded in country  $r$ . Thus relative demand for labour is decreasing in country  $r$ , which implies relatively more demand for skilled workers, and the relative wage rate is rising. On the other hand, if sector  $k$  is relatively skill-intensive in country  $r$  (irrespective of the ranking in the innovating country  $c$ ) the relative wage rate will fall.

<sup>21</sup>For  $\sigma = 1$ , i.e. a Cobb-Douglas production function, the equilibrium wage rates can be calculated directly for each country.

<sup>22</sup>Technical progress only has an effect on the equilibrium exchange rates which have to change accordingly to restore the external equilibrium. Of course, this implies welfare effects as terms of trades are changing; these are, however, beyond the scope of this paper.

The former case could explain the rising wage premium in the advanced economies (which in this case corresponds to the domestic economy) in an international framework: If some countries are catching up quite rapidly in the labour-intensive sectors (or in the labour-intensive segments of high-tech sectors such as electronics, machinery, etc. which are satisfying the condition) the relative wage rate in the advanced economies will rise due to specialization effects. On the other hand, a rising skill premium in the less advanced economies (which are now interpreted as the domestic economy) could only be explained by concentration of technical change in the labour-intensive sectors of the advanced economies.

TECHNICAL CHANGE IN THE DOMESTIC ECONOMY. Taking derivatives with respect to  $A_k^r$  yields the condition for a rising relative wage in this economy given by

$$(\tilde{\zeta} - 1)\theta_k^r > \omega^r h^r (\tilde{\zeta} - 1)$$

with  $\tilde{\zeta} := \varphi - \varsigma\mu_k^{rr}(\varphi - 1) = \varphi(1 - \varsigma\mu_k^{rr}) + \varsigma\mu_k^{rr}$  (see appendix); the latter expression depends on the elasticities of substitution  $\varsigma$  and  $\varphi$  and the expenditure share for the composite good  $k$  to which the sector belongs in the own economy (see appendix for an exact definition of  $\mu_k^{rr}$ ). We have to distinguish two cases: The condition  $\tilde{\zeta} > 1$  is satisfied if  $0 < \varsigma < 1/\mu_k^{rr}$ , i.e. the elasticity of substitution between goods  $\varsigma$  is not too large (although it may be considerably larger than one if  $\mu_k^{rr}$  is small). In the other case,  $\tilde{\zeta} < 1$  implies that  $\varsigma > 1/\mu_k^{rr}$ ; this is more likely to be satisfied the larger the expenditure share of demand in the home country.

Let us start with the first case: If  $\tilde{\zeta} > 1$  this corresponds to the case  $\varsigma > 1$  in the closed economy as the condition becomes  $\theta_k^r > \omega^r h^r$ . If total factor productivity is rising in a skill-intensive sector of the domestic economy  $r$ , the relative wage rate is rising as the economy is attracting demand in this skill-intensive sector and the elasticities of substitution are high. Thus, concentration of technical change in the skill-intensive sectors explains the rising wage premium in the advanced countries (as the domestic economies) also in cases where international specialization effects are taken into account as well. However, this may be at odds with some evidence that the wage premium is rising in the less advanced economies as well, as - given the discussion above - relative wages in the domestic and the foreign economies would move in opposite directions (if the innovating sector is characterized as either skill- or labour-intensive in all economies). When considering the catching-up economies as the domestic economy, the skill premium would rise if catching-up is concentrated in the skill-intensive sectors; however, from this one should then expect a declining relative wage in the advanced economies.

What happens if  $\tilde{\zeta} < 1$ , which may be relevant if the substitution effects are very strong? The condition becomes  $\theta_k^r < \omega^r h^r$ , which formally corresponds to the case  $\varsigma < 1$  in the closed economy; i.e. the relative wage is rising if technical change takes place in a labour-intensive sector. However, the interpretation is different: If total factor productivity is increasing in a skill-intensive sector it attracts quite a lot of demand; this raises demand for skills, which implies a higher relative wage. However, if this effect is very strong, costs in the skill-intensive sectors are rising as well; given the strong substitution effects, demand shifts to the labour-intensive goods in general and demand for skill-intensive goods shifts from the domestic (innovating) economy to the foreign economy. The overall effect would then be a decrease in the relative wage of the innovating economy.

Note that in this case the relative wage rate is going in the same direction in both the domestic and the foreign economies (again, if the innovating sector is characterized as either skill- or labour-intensive in all economies). However, to explain the rising wage premium technical change would have to be concentrated

in the labour-intensive industries. Thus a potential explanation for a rising skill premium in both types of countries would be that catching-up is concentrated in the labour-intensive sectors, which would raise the wage premium in the less advanced economies (which in this case has be interpreted as the domestic economy). Above we have already argued that catching-up in labour-intensive sectors leads to a higher relative wage in the advanced countries.<sup>23</sup>

### 3.2.3 Factor- and sector-biased technical change

Let us now consider changes in the skill-augmenting parameters in the domestic and the foreign economy, i.e.  $a_{kS}^r$  or  $a_{kS}^c$ , respectively.

**TECHNICAL CHANGE IN A FOREIGN ECONOMY.** We start with the effects of SATC taking place in the foreign economy and study the effects on the domestic economy. The first derivative with respect to  $a_{kS}^c$  yields again the condition  $\theta_k^r < \omega^r h^r$  (see appendix), i.e. the same condition as in the case of an increase in total factor productivity in a foreign country. As above, this condition does not depend on the elasticities of substitution in demand or production as technical changes are transmitted only via changes in relative prices. The same interpretation as above applies.

**TECHNICAL CHANGE IN THE DOMESTIC ECONOMY.** Finally let us investigate the case of SATC in the domestic country  $r$ . In the appendix we show that the first derivative with respect to  $a_{kS}^r$  yields the following condition for a rising relative wage rate<sup>24</sup>

$$(\sigma - 1) + (\tilde{\zeta} - 1)\theta_k^r > \omega^r h^r (\tilde{\zeta} - \sigma).$$

Similar to the case of the closed economy we calculate the critical value as  $\theta_*^r = \omega^r h^r \frac{\tilde{\zeta} - \sigma}{\tilde{\zeta} - 1} - \frac{\sigma - 1}{\tilde{\zeta} - 1}$  which is the same expression for the closed economy with  $\varsigma$  replaced by  $\tilde{\zeta}$ . Again we have to distinguish two cases, namely  $\tilde{\zeta} < 1$  and  $\tilde{\zeta} > 1$ .<sup>25</sup> In figure 3.3 we have again drawn the relationships for different combinations of  $\sigma$  and  $\tilde{\zeta}$ . Let us discuss the various cases in more detail.

**FIRST CASE:** Let us first dicuss the case  $\tilde{\zeta} > 1$  which implies  $\varsigma < 1/\mu_k^{rr}$  where  $1/\mu_k^{rr} \geq 1$  (this corresponds to the panel 2 of figure 3.3). If this condition is satisfied the relative wage rate is rising if  $\theta_k^r > \theta_*^r$  and falling otherwise. This exactly corresponds to the case of  $\varsigma > 1$  in the closed economy case and the same interpretation holds as well (see **THIRD CASE** in section 2.2.2 above). Let us only discuss the cases where SATC is skill-using, i.e. the lines depicted  $1 < \sigma < \tilde{\zeta}$ ,  $\sigma = \tilde{\zeta}$  and  $\sigma > \tilde{\zeta}$  in panel 2 of figure 3.3. Skill-biased technical progress in a large range of sectors of the advanced economies which are not too labour-intensive results in a rising skill premium in the advanced economies. If  $\sigma \geq \tilde{\zeta}$ , which could be satisfied for small values of  $\varsigma$ , SATC in any sector would imply a rising skill premium. (This would imply that the sector bias of SATC does not matter at all and only the factor bias is important.)

Again the rising skill premium in both the advanced as well as the less advanced economies can be explained. Assume that the less advanced economies are experiencing a skill-biased catching up process in the sectors with skill-intensities above the critical value (again this could include skill-intensive as well as labour-intensive sectors according to our definition). In this case the skill premium in the less

<sup>23</sup>Note that the condition  $\tilde{\zeta} < 1$  can be satisfied for reasonable values of the elasticities. If the catching-up economies are specialized in the labour-intensive goods the share  $\mu_k^{rr}$  is relatively high. For instance, if this would be 0.75, the elasticity of substitution  $\varsigma$  would have to be larger than 1.333. In CGE models often parameter values of 2.5 are used.

<sup>24</sup>Similar to above, the condition under the assumption of a Cobb-Douglas production must be derived separately. But the condition becomes  $(\tilde{\zeta} - 1)\theta_k^r > \omega^r h^r (\tilde{\zeta} - 1)$  which is exactly the condition above with  $\sigma = 1$ .

<sup>25</sup>The case  $\tilde{\zeta} = 1$  corresponds to the case  $\varphi = 1$  discussed above.

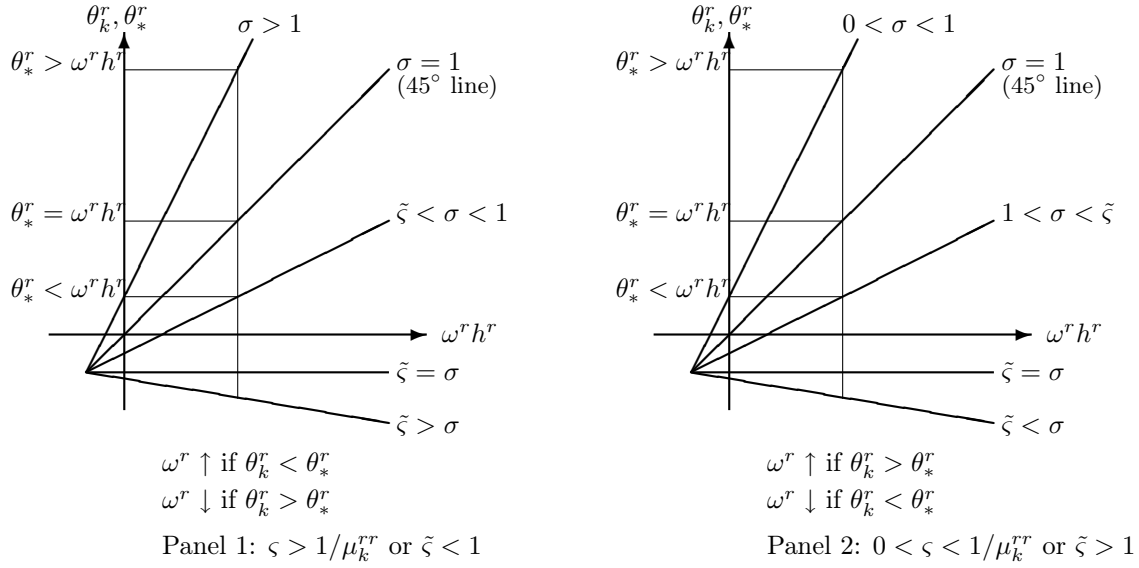


Figure 3.3: Critical values

advanced economy would rise if technical change is not concentrated in the most labour-intensive sectors. If further catching-up is concentrated in the sectors (or segments of sectors) which are labour-intensive in the advanced economies, the skill premium would rise there as well.

SECOND CASE: In the second case (see panel 1 of figure 3.3)  $\tilde{\zeta} < 1$ , which requires  $\zeta > 1/\mu_k^{rr} > 1$  (the elasticity of substitution between goods  $\zeta$  has now to be larger than a critical value depending on  $\mu_k^{rr}$ ; in particular, this implies that it must be larger than one). The condition means that the substitution effect in demand has to be sufficiently strong. In this case the relative wage rate is rising if  $\theta_k^r < \theta_*^r$  and falling otherwise. Formally this corresponds to the case  $\zeta < 1$  for the closed economy. The interpretation of this result is analogous to the case of a change in total factor productivity already discussed above; the line depicted with  $\sigma = 1$  in panel 1 of figure 3.3 exactly corresponds to that case. If SATC has a skill-using character (which requires  $\sigma > 1$ ) this line rotates counterclockwise and the range of sectors where SATC leads to a rising skill premium becomes larger. Only SATC in the very skill-intensive sectors would lead to a falling skill premium.

Similar to the case  $\tilde{\zeta} < 1$  above this could be used as an explanation for a rising skill premium in both types of countries. Let us just discuss the case of catching-up. Catching-up of a skill-biased character in any sector with the exception of the very skill-intensive ones would lead to a rising skill premium in the less developed countries. Again, if technical change is concentrated in sectors which are labour-intensive in the advanced economies, the skill premium in the latter countries would rise as well.

Let us summarize these results: Empirical evidence shows that skill-biased technical change has been concentrated in the skill-intensive sectors in the advanced economies. This can explain the rising skill premium in the advanced economies; however, if these sectors are the skill-intensive sectors in the less advanced economies one would expect a decrease of the relative wage there. This latter result may not be supported by the empirical evidence that the skill premium has been rising in these countries as well.

However, skill-biased catching-up of less advanced economies may lead to a rising relative wage in both the less advanced catching-up and the advanced economies.<sup>26</sup>

## 4 Concluding remarks

In this paper we introduced a framework allowing for any discrete number of sectors and countries and two factors to investigate the effects different types of technical change on relative factor prices in the innovating as well as the other economies. From a methodological point of view the model allows for differences in technology by country and sector (i.e. total factor productivity, skill-augmenting parameters and - in a slightly more general version - even different elasticities of substitution) and differences in factor endowment structures across countries and thus integrates technology-based with factor endowment-based views on trade. The model is particularly interesting for studying the effects of catching-up processes of less advanced economies concentrated in particular sectors, the effects of (defensive) innovation in advanced economies, changes in factor endowments, etc. We were able to derive necessary and sufficient conditions under which technical change causes an increase in the relative wage rate. These conditions are determined by the sectoral skill intensity, the elasticities of substitution in production and demand, relative endowment and the equilibrium wage rate. The analytical results are useful not only for modelling and computational research but also for further empirical studies. Furthermore, the conditions allow for a straightforward economic interpretation and shed light on the cases which are ambiguous in the existing literature so far.

We have also provided interpretations of the existing empirical studies, with special emphasis on the sector- versus factor-biased hypotheses. In general, the stronger the skill-biased character of technical change (i.e. the more skill-using technical change is in the Hicksian sense) the larger is the range of sectors where innovation leads to a rising skill premium. This sheds a different light on the two hypotheses: Skill-biased technical change may not be heavily concentrated in a particular subset of sectors (either the most labour or the most skill-intensive ones, depending on the elasticity of substitution in demand) to explain the rising skill premium. This may be the reason why most empirical studies find the factor bias being more important than the sector bias. Furthermore, the phenomenon of catching-up with a skill-biased character of less advanced economies in a wide range of sectors can explain that relative wages of skilled workers are rising in both the advanced and the less advanced economies, which provides an alternative hypothesis on this issue.

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<sup>26</sup>In the case  $\zeta < 1$ , panel 2 of figure 3.3, SATC may not be concentrated in the least labour-intensive sectors, whereas in the case  $\zeta > 1$  it may not be concentrated in the most skill-intensive sectors.



# A Appendix

## A.1 Cost-reducing criterion

Before turning to the model, let us analyse whether the various forms of technical progress discussed above are in fact cost-reducing at constant factor prices, which is used as a criterion for whether the innovation is actually introduced. If this were not the case, the particular change in the coefficients may not take place at all. The criterion, however, does not rule out that the costs of production of a particular good will rise once all general equilibrium adjustments have taken place.

Straightforward cost-minimization yields that the costs of production are given by

$$c_i = A_i^{-1} [\alpha_{iL}^\sigma a_{iL}^{\sigma-1} + \omega^{1-\sigma} \alpha_{iS}^\sigma a_{iS}^{\sigma-1}]^{\frac{1}{1-\sigma}}$$

for  $\sigma \neq 1$ . The first derivative of  $c_i$  with respect to a particular technical parameter evaluated at constant factor prices must be negative to meet the cost-reducing criterion. Obviously  $\partial c_i / \partial A_i < 0$ , i.e. total factor productivity growth is cost-reducing. Taking the derivative with respect to  $a_{iS}$  yields

$$\frac{dc_i}{da_{iS}} = A_i^{-1} \frac{1}{1-\sigma} [\alpha_{iL}^\sigma a_{iL}^{\sigma-1} + \omega^{1-\sigma} \alpha_{iS}^\sigma a_{iS}^{\sigma-1}]^{\frac{1}{1-\sigma}-1} \omega^{1-\sigma} \alpha_{iS}^\sigma (\sigma-1) a_{iS}^{\sigma-2} < 0$$

as  $(\sigma-1)/(1-\sigma) = -1$ . For  $a_{iL}$  this can be shown analogously. Thus, total factor productivity growth and factor-augmenting technical progress are cost-reducing in any case. This includes the case of a change in the parameters  $a_{iz}$  with  $\sigma = 1$ .

## A.2 Closed economy

The equilibrium condition is given in the text. Taking derivatives with respect to  $A_k$  the derivation of the condition is straightforward; thus we only discuss changes in skill-augmenting parameters. The first derivatives of (2.5) with respect to  $a_{kz}$  are given by

$$\begin{aligned} \frac{\partial D}{\partial a_{kS}} &= \omega^{-1} \beta_k^\zeta A_k^{\zeta-1} \omega^{1-\sigma} \alpha_{kS}^\sigma a_{kS}^{\sigma-2} B_k^{\frac{\sigma-\zeta}{1-\sigma}} [(\sigma-1) - (\sigma-\zeta)\theta_{kS}] \\ \frac{\partial S}{\partial a_{kS}} &= h \beta_k^\zeta A_k^{\zeta-1} \omega^{1-\sigma} \alpha_{kS}^\sigma a_{kS}^{\sigma-2} B_k^{\frac{\sigma-\zeta}{1-\sigma}} (\zeta-\sigma)\theta_{kL} \end{aligned}$$

and the condition  $\frac{\partial D}{\partial a_{kS}} > \frac{\partial S}{\partial a_{kS}}$  can be expressed as:

$$\begin{aligned} (\sigma-1) - (\sigma-\zeta)\theta_{kS} &> \omega h (\zeta-\sigma)\theta_{kL} \\ (\sigma-1)\theta_{kS} + (\sigma-1)\theta_{kL} - (\sigma-\zeta)\theta_{kS} &> \omega h (\zeta-\sigma)\theta_{kL} \\ (\sigma-1)\theta_k + (\sigma-1) - (\sigma-\zeta)\theta_k &> \omega h (\zeta-\sigma) \end{aligned}$$

Rearranging the last term gives the expression used in the text (see section 2.2.2).

The first derivative of the critical value  $\theta_* = \omega h \frac{\zeta-\sigma}{\zeta-1} - \frac{\sigma-1}{\zeta-1}$  with respect to  $\zeta$  is  $\frac{\partial \theta_*}{\partial \zeta} = \omega h \frac{\sigma-1}{(\zeta-1)^2} + \frac{\sigma-1}{(\zeta-1)^2}$  which is negative for  $\sigma < 1$ , zero if  $\sigma = 1$  and positive for  $\sigma > 1$ . The derivative with respect to  $\sigma$  is  $\frac{\partial \theta_*}{\partial \sigma} = -\omega h \frac{1}{\zeta-1} - \frac{1}{\zeta-1}$  which is negative for  $\zeta > 1$  and positive for  $\zeta < 1$ .

## A.3 Trading economies

### A.3.1 Demand structure

In the following we use a nested CES function. We allow for different elasticities of substitution in demand across goods  $\varsigma$  and for a particular good  $k$  across countries  $\varphi$ . Assuming that the utility function in country  $c$  is given by  $U^c = (\sum_j \beta_j (Q_j^c)^\varrho)^{1/\varrho}$  with  $\sum_j \beta_j = 1$  where  $Q_j^c = (\sum_r \delta_j^r (q_j^r)^\phi)^{1/\phi}$  is a composite good (composed of the goods  $j$  produced in the own and the other countries) with  $\sum_r \delta_j^r = 1$  for all  $j$ . As price index for the composite good we use  $P_j^{(t)} = (\sum_s (\epsilon^{ts} p_s^s)^{1-\varphi} (\delta_i^s)^\varphi)$  where the exchange rates  $\epsilon^{ts}$  express all prices in a common currency of country  $t$ . This results from the optimization problem. As we assume equal preferences (expressed in the equality of parameters)<sup>27</sup> the price index is equal for all countries; the superscript in brackets indicates that the index is expressed in currency of country  $t$ . Demand for the composite good is given by  $Q_j^c = Y^c \frac{(P_j^{(t)})^{-\varsigma} \beta_j^\varsigma}{\sum_k (P_k^{(t)})^{1-\varsigma} \beta_k^\varsigma}$ . Simple manipulation shows that in country  $c$  the share of income spent on (the composite) good  $j$  is  $\gamma_j^c = \frac{(P_j^{(t)})^{1-\varsigma} \beta_j^\varsigma}{\sum_k (P_k^{(t)})^{1-\varsigma} \beta_k^\varsigma}$  which satisfies the constraint  $\sum_j \gamma_j^c = 1$ . The nominal expenditure share in country  $c$  for good  $k$  (of the composite good) demanded in country  $r$  is  $\mu_k^{rc} = \frac{(\epsilon^{tr} p_k^r)^{1-\varphi} (\delta^r)^\varphi}{\sum_s (\epsilon^{ts} p_s^s)^{1-\varphi} (\delta^s)^\varphi} = \frac{(\epsilon^{tr} p_k^r)^{1-\varphi} (\delta^r)^\varphi}{P_k^{(t)}}$  which satisfies  $\sum_r \mu_k^{rc} = 1$ ; we set  $P^{(t)} = \sum_k (P_k^{(t)})^{1-\varsigma} \beta_k^\varsigma$ . Multiplication of these two terms gives the overall expenditure shares

$$\gamma_j^{rc} = \frac{(\epsilon^{tr} p_k^r)^{1-\varphi} (\delta^r)^\varphi (P_j^{(t)})^{1-\varsigma} \beta_j^\varsigma}{P_j^{(t)} P^{(t)}}.$$

These shares also satisfy the constraint  $\sum_{j,r} \gamma_j^{cr} = 1$  for all countries  $c = 1, \dots, C$ . For the special case  $\varphi = 1$  the expenditure shares become  $\gamma_j^c = \beta_j^\varsigma / \sum_k \beta_k^\varsigma$  and  $\mu_j^{rc} = \delta^r$ . Multiplication gives  $\gamma_j^{rc} = \gamma_j^c \mu_j^{rc} = (\beta_j^\varsigma / \sum_k \beta_k^\varsigma) \delta_j^r$ , i.e. a constant.

### A.3.2 Equilibrium condition

As already stated in the text the employment rates are given by  $E_S^r = \frac{1}{h_S^r} \sum_{c=1}^C \sum_{i=1}^N \frac{\gamma_i^{rc} (\epsilon^{rc} Y^c)}{p_i^r} \tilde{a}_{iS}^r$  and  $E_L^r = \frac{1}{h_L^r} h^r \sum_{c=1}^C \sum_{i=1}^N \frac{\gamma_i^{rc} (\epsilon^{rc} Y^c)}{p_i^r} \tilde{a}_{iL}^r$ , respectively. For trading economies a further condition is given by the balance of payments equilibrium. The balance of payments ( $BoP^{rc}$ ) for a country pair is the value of exports of country  $r$  to country  $c$  minus the value of imports of country  $c$  from country  $r$  (i.e. the net exports of country  $r$ ). Thus the BoP between countries  $r$  and  $c$  is given by  $BoP^{rc} = \sum_{j=1}^N \epsilon^{rc} \gamma_j^{rc} Y^c - \sum_{j=1}^N \gamma_j^{cr} Y^r = \epsilon^{rc} Y^c \sum_{j=1}^N \gamma_j^{rc} - Y^r \sum_{j=1}^N \gamma_j^{cr}$  which is here expressed in currency of country  $r$ . (A decrease in  $\epsilon^{rc}$  is a depreciation of the currency of country  $c$  and an appreciation of the currency of country  $r$ .) A balance of payments equilibrium requires  $BoP^{rc} = 0$  and thus  $\epsilon^{rc} Y^c \sum_{j=1}^N \gamma_j^{rc} = Y^r \sum_{j=1}^N \gamma_j^{cr}$  or  $\epsilon^{rc} = \frac{Y^r \sum_{j=1}^N \gamma_j^{cr}}{Y^c \sum_{j=1}^N \gamma_j^{rc}}$ . An analysis of this international equilibrium which requires  $BoP^{rc} = 0$  for all country pairs  $r, c$  and  $E^r(\omega^r) = 0$  for all countries  $r$  with respect to the existence, uniqueness and stability of an equilibrium is not provided here as it would go beyond the scope of this paper. Let us only note the results from numerical simulations which emphasize the assumption of a balance-of-payments condition: First, an overall equilibrium exists for a wide range of parameter values. Second, the results do not change qualitatively when allowing for changes in the exchange rates and relative wage rates in all countries

<sup>27</sup>Here again one may use a slightly more general formulation which would allow for country-specific utility functions. However, the analysis then becomes more tedious and thus we stick to this simple case. Further, this is also in line with the recent literature assuming equal preferences for all countries.

to take place; on the other hand, the results do not change qualitatively when allowing for balance-of-payments surpluses or deficits. Let us finally note that the relationship between exchange rate movements and the relative wage rates are non-linear. In particular, a change in the exchange rate thus also has structural implications (i.e. on relative prices, output and factor prices).

Inserting the balance-of-payments equilibrium condition in the employment rates above the equilibrium condition can be derived (see equation (3.7) in the text). Inserting for expenditure shares, prices and input coefficients, the equilibrium condition becomes

$$\begin{aligned} (\omega^r)^{-\sigma} \sum_{i=1}^N \beta_i^\varsigma (A_i^r)^{\varphi-1} (\alpha_{iS}^r)^\sigma (a_{iS}^r)^{\sigma-1} (B_i^r)^{\frac{\sigma-\varphi}{1-\sigma}} \left( \sum_s (\epsilon^{ts})^{1-\varphi} (\delta^s)^\varphi (A_i^s)^{\varphi-1} (B_i^s)^{\frac{1-\varphi}{1-\sigma}} \right)^{-\varsigma} \\ = \\ h^r \sum_{i=1}^N \beta_i^\varsigma (A_i^r)^{\varphi-1} (\alpha_{iL}^r)^\sigma (a_{iL}^r)^{\sigma-1} (B_i^r)^{\frac{\sigma-\varphi}{1-\sigma}} \left( \sum_s (\epsilon^{ts})^{1-\varphi} (\delta^s)^\varphi (A_i^s)^{\varphi-1} (B_i^s)^{\frac{1-\varphi}{1-\sigma}} \right)^{-\varsigma} \end{aligned}$$

For changes in  $A_k^r$  or  $A_k^c$  the same analysis applies as for the integrated economy with  $\varsigma > 1$ . Thus we only have to consider changes in the skill-augmenting parameters  $a_{kS}^r$  or  $a_{kS}^c$ .

### A.3.3 Changes in the factor-augmenting parameters

We here only show the most important steps in deriving the conditions with respect to the parameter  $a_{kS}$ . The calculations for changes in total factor productivity growth are similar. We have to distinguish whether technical change is taking place in the domestic or a foreign economy.

DOMESTIC ECONOMY: Taking the first derivative with respect to  $a_{kS}^r$ , using the condition for a rising relative wage rate and simplifying yields

$$\theta_k [(\sigma - 1) + (\varphi - \sigma)\theta_{kS}^r - \varsigma(\varphi - 1)\theta_{kS}^r \mu_k^{rr}] > \omega^r h^r [(\varphi - \sigma)\theta_{kS}^r - \varsigma(\varphi - 1)\theta_{kS}^r \mu_k^{rr}].$$

Dividing this expression by  $\theta_{kS}^r$  and using  $\theta_k^r / \theta_{kS}^r = \theta_k^r + 1$  (note that  $0 < \theta_{kz} < 1$ ) yields

$$(\sigma - 1) + \theta_k^r [(\varphi - 1) - \varsigma(\varphi - 1)\mu_k^{rr}] > \omega^r h^r [(\varphi - \sigma) - \varsigma(\varphi - 1)\mu_k^{rr}].$$

Using the definition of  $\tilde{\zeta} := \varphi - \varsigma\mu_k^{rr}(\varphi - 1)$  gives the condition stated in the text. For a change in total factor productivity  $A_k$  the expression becomes  $\theta_k^r [(\varphi - 1) - \varsigma(\varphi - 1)\mu_k^{rr}] > \omega^r h^r [(\varphi - 1) - \varsigma(\varphi - 1)\mu_k^{rr}]$ . This is a special case of the former expression as under  $\sigma = 1$  a change in the skill-augmenting parameter is Hicks neutral. Again using the definition of  $\tilde{\zeta}$  yields the condition.

FOREIGN ECONOMY: Finally, let us consider the case of skill-augmenting innovation taking place in sector  $k$  of foreign country  $c$ . Taking the first derivative with respect to  $a_{kS}^c$  and simplifying yields

$$(\omega^r)^{-\sigma} (\alpha_{kS}^r)^\sigma (a_{kS}^r)^{\sigma-1} (1 - \varphi) > h^r (\alpha_{kL}^r)^\sigma (a_{kL}^r)^{\sigma-1} (1 - \varphi).$$

Using the assumption  $\varphi > 1$  and the definition of  $\theta_k$  yields

$$\theta_k^r < \omega^r h^r.$$

## References

- Acemoglu, D. (2002a). Directed technical change. *Review of Economic Studies* 69, 781–809.
- Acemoglu, D. (2002b). Technical change, inequality and the labour market. *Journal of Economic Literature* XL(1), 7–72.
- Davis, D. (1998). Technology, unemployment, and relative wages in a global economy. *European Economic Review* 42, 1613–1633.
- Dinopolous, E., C. Syropoulos, and B. Xu (2001). Intra-industry trade and wage-income inequality. *Working Paper, University of Florida*.
- Dixit, A. and V. Norman (1980). *Theory of International Trade*. Cambridge, GB: Cambridge University Press.
- Dornbusch, R., S. Fischer, and P. Samuelson (1977). Comparative advantage, trade, and payments in a Ricardian model with a continuum of goods. *Economic Journal* 67, 823–839.
- Dornbusch, R., S. Fischer, and P. Samuelson (1980). Heckscher-Ohlin trade theory with a continuum of goods. *Quarterly Journal of Economics* 95, 203–224.
- Feenstra, R. and G. Hanson (2001). Global production sharing and rising inequality: A survey of trade and wages. *NBER Working Paper 8372*.
- Findlay, R. and H. Grubert (1959). Factor intensities, technological progress, and the terms of trade. *Oxford Economic Papers* 11, 111–121.
- Harrod, R. (1942). *Toward a Dynamic Economics: Some Recent Developments of Economic Theory and their Application to Policy*. London: Macmillan.
- Haskel, J. and M. Slaughter (2002). Does the sector bias of skill-biased technical change explain changing skill premia? *European Economic Review* 46, 1757–1783.
- Hicks, J. (1932). *The Theory of Wages*. London: Macmillan.
- Krugman, P. (2000). Technology, trade, and factor prices. *Journal of International Economics* 50, 51–71.
- Landesmann, M. and R. Stehrer (2001). Convergence patterns and switchovers in comparative advantage. *Structural Change and Economic Dynamics* 12, 399–423.
- Leamer, E. (1998). In search of Stolper-Samuelson linkages between international trade and lower wages. In S. Collins (Ed.), *Imports, Exports, and the American Worker*. Brookings Institution, Washington, DC.
- Neary, J. (2003). Globalization and market structure. *Journal of the European Economic Association* 1, 245–271.
- Robinson, J. (1938). The classification of invention. *Review of Economic Studies* 5, 139–142.

- Romalis, J. (2004). Factor proportions and the structure of commodity trade. *American Economic Review* 94(1), 67–97.
- Stehrer, R. and J. Woerz (2003). Technological convergence and trade patterns. *Weltwirtschaftliches Archiv/Review of World Economics* 139(2), 1–28.
- Trefler, D. (1993). International factor price differences: Leontief was right! *Journal of Political Economy* 101(6), 961–987.
- Uzawa, H. (1961). Neutral inventions and the stability of growth equilibrium. *Review of Economic Studies* 28, 117–124.
- Wood, A. (1997). *North-South Trade: Employment and Inequality. Changing Fortunes in a Skill-Driven World*. Oxford: Clarendon Press.
- Xu, B. (2001). Factor bias, sector bias, and the effects of technical progress on relative factor prices. *Journal of International Economics* 54, 5–25.
- Zhu, S. and D. Trefler (2005). Trade and inequality in developing countries: A general equilibrium analysis. *Journal of International Economics* 65(1), 21–48.